Student details

Name: _____

Mark: _____

2023

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 6 – 13

90 marks

- Attempt Questions 11 – 31
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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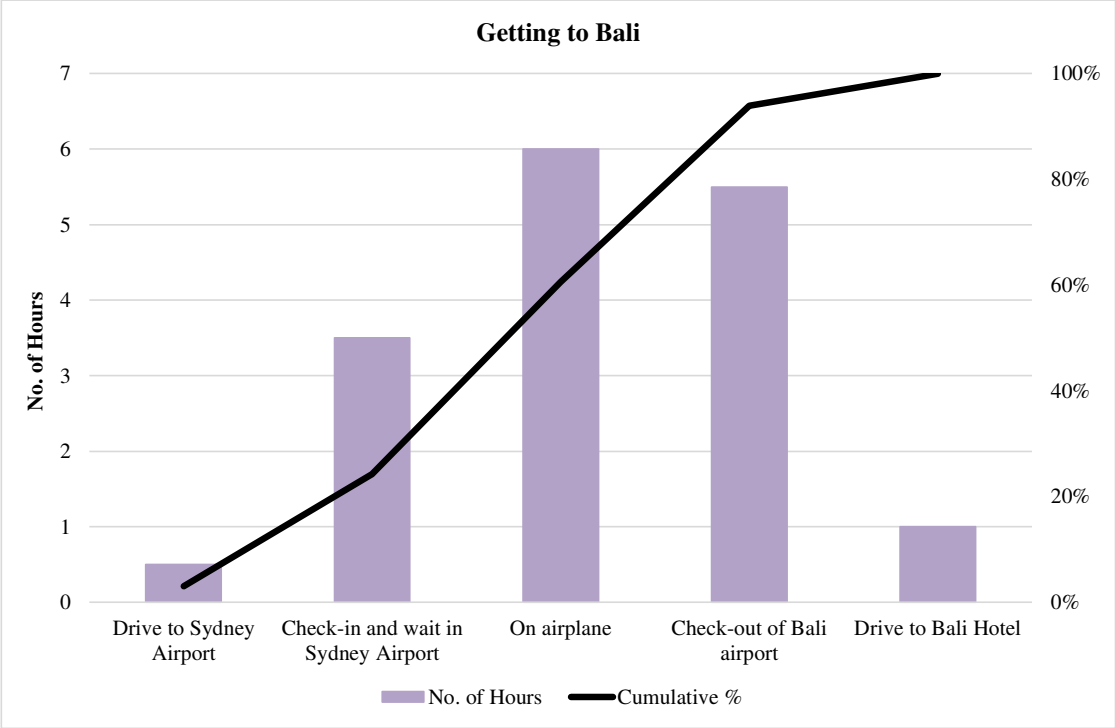
1 The following pareto chart shows the time Ricky's family spent getting to Bali:



Approximately what percentage of time was spent checking-out of Bali airport?

(A) 6%

(B) 33%

(C) 84%

(D) 94%

2 What is the limiting sum of the series $\frac{5}{3} + \frac{2}{3} + \frac{4}{15} + \ldots$?

(A) 2

(B) $\frac{25}{3}$

(C) $\frac{25}{6}$

(D) $\frac{25}{9}$

3 Which of the following best describes the curve $x = -\sqrt{6 - y^2}$

(A) One-to-one function

(B) Many-to-one function

(C) One-to-many relation

(D) Many-to-many relation

4 What is the derivative of $\log_e\left(\frac{3x + 4}{4x + 1}\right)$?

(A) $\frac{4x + 1}{3x + 4}$

(B) $-\frac{13}{(3x+4)(4x+1)}$

(C) $\frac{(3x+4)(4x+1)}{12}$

(D) $\frac{3}{4}\log_e\left(\frac{3x + 4}{4x + 1}\right)$

5 Consider the function $f(x) = \frac{3}{x} + 6$. Which of the following statements is true?

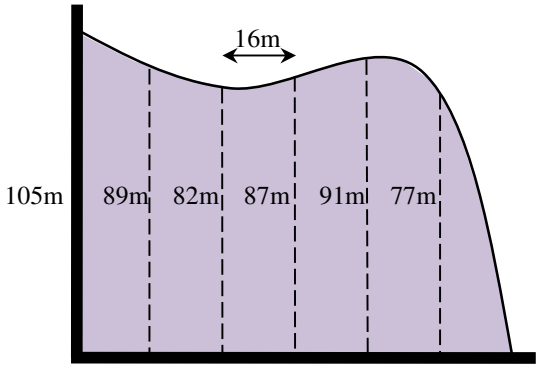
(A) $f(x)$ is an even function.

(B) $f(x)$ has one point of discontinuity.

(C) $f(x)$ has an asymptote at $x = 3$.

(D) None of the above are true.

6 For a school geography project, Kan had to estimate the area of his local park. The following diagram shows an aerial view of Kan's local park with key measurements measured at equal distances of 16m.



Using the Trapezoidal rule, what is the approximate area of the park?

(A) 957 m^2

(B) 880 m^2

(C) 7040 m^2

(D) 7656 m^2

7 What is the amplitude and period of the function $5y = 2\sin\left(\frac{8\pi x + 1}{3}\right)$?

(A) Amplitude = 2; Period = 8π

(B) Amplitude = $\frac{5}{2}$; Period = $\frac{8\pi}{3}$

(C) Amplitude = 5; Period = $\frac{1}{3}$

(D) Amplitude = $\frac{2}{5}$; Period = $\frac{3}{4}$

8 For a particular function $y = f(x)$, the curve of the second derivative graph can be described as a concave-down parabola with vertex at (0, –3). Which of the following statements is false?

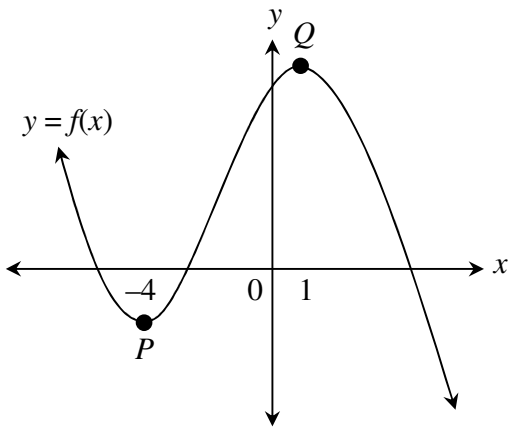
(A) The function $y = f(x)$ is a quartic function.

(B) The function $y = f(x)$ may have at least one stationary point.

(C) The function $y = f(x)$ may have at least one inflexion point.

(D) The function $y = f(x)$ may have an x-intercept at the origin.

9 Consider $y = f(x)$ where a local minimum P can be found at $x = -4$ and a local maximum Q can be found at $x = 1$, as shown in the diagram.



Which of the following lists the values of $f''(1)$, $f'(-4)$ and $f(1)$ in ascending order?

(A) $f''(1), f'(-4), f(1)$

(B) $f'(-4), f(1), f''(1)$

(C) $f(1), f'(-4), f''(1)$

(D) $f'(-4), f''(1), f(1)$

10 What are the values for a and b for which $\int_a^b \sin x \, dx < \int_a^b \cos x \, dx$ is true?

(A) $a = 0$ and $b = 2\pi$

(B) $a = 0$ and $b = \pi$

(C) $a = \frac{\pi}{2}$ and $b = \frac{3\pi}{2}$

(D) $a = \pi$ and $b = 2\pi$

Section II

90 marks
Attempt Questions 11–32

In Questions 11–32, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Solve for x and y: $3x + y = 10$
$x + 5y = -6$ **2**

Question 12

Find the sum of the first 27 terms in the series $8 + 11 + 14 + \ldots$ **2**

Question 13

Find the equation of the normal to the curve $y = (2x - 3)^3$ at $x = 2$. **3**

Question 14

Differentiate the following with respect to x:

(a) $y = (4x - 5)^8$. **1**

(b) $y = \sqrt{e^{3x} - 2}$. **2**

(c) $y = \dfrac{\cos 5x}{x^2 + 3}$. **2**

(d) $y = x \log_e (3x)$. **2**

Question 15

A school has two accelerated biology classes – BIO_1 containing 15 students and BIO_2 containing 16 students. For the first assessment, the average for BIO_1 was 86.4% while the combined average across both classes was 88.3%. **2**

Find the average of BIO_2 rounding your solution to one decimal place.

Question 16

Find:

(a) $\int \frac{3x^3}{5x^4 + 2}\, dx$. **1**

(b) $\int \sin(\pi x) e^{\cos(\pi x)}\, dx$. **1**

(c) $\int_0^2 \left(e^x + 3\right)^2\, dx$. **2**

Question 17

A geometric progression has 2 as the sixth term and $\frac{2}{27}$ as the ninth term.

(a) Find the value of the first term and the common ratio. **2**

(b) Find the value of the 11^{th} term. **1**

(c) Find the sum of the first 9 terms in the series. **1**

Question 18

Solve for x: $9^x + 27 = 4(3^{x+1})$. **3**

Question 19

(a) Prove the identity: $\left(\text{cosec}^2\theta - 1\right)\sin^2\theta = \cos^2\theta$. **2**

(b) Hence, or otherwise, find the value of θ for $-\pi \le \theta \le \pi$: $\left(\text{cosec}^2\theta - 1\right)\sin^2\theta = \frac{3}{4}$ **3**

Question 20

A copper mine was established near a rural town where copper (in tonnes) was extracted according to the rate:

$$C = 420e^{-kt}$$

where k is a positive constant and t is in years.

(a) If 75% of the copper was remaining in the mine after 3 years, show that: **2**

$$k = \frac{1}{3}\log_e \frac{4}{3}.$$

(b) At what rate is the copper decreasing at after 4 years? Round your solution to the nearest whole number. **2**

(c) The copper mine will be shut down when the copper in the mine is less than 30 tonnes. After how many years will the copper mine be shut down? Round your solution to one decimal place. **2**

Question 21

Consider the curve $y = 4\log_e x + 6 - x$ for $1 \le x \le 10$.

(a) Find any stationary point(s) and determine their nature. **3**

(b) Comment on the concavity of the function, providing evidence. **2**

(c) Hence, or otherwise, sketch the curve $y = 4\log_e x + 6 - x$ on a number plane, showing all key features. **2**

Question 22

If $\log_8(x + 2) - \log_8(x - 1) = \frac{4}{3}$, find the value of x. **2**

Question 23

Deaths related to guns being available to the public is a common topic being debated. The following table summarises a survey across 10 countries where the number of guns held by civilians was compared to the number of gun-related deaths in the last calendar year.

Country	No. of guns held in '000 (G)	No. of gun-related deaths (D)
A	1,525	325
B	82	2
C	252	5
D	845	98
E	16,225	875
F	572	42
G	35	1
H	611	15
I	876,400	5,667
J	58	8

By using the table above, and rounding your solution to two decimal places where required:

(a) State the independent variable. **1**

(b) Determine the value of Pearson's correlation coefficient (r). **1**

(c) Using (b), describe the strength of the correlation between D and G. **1**

(d) Find the 'line of best fit', stating your solution as $D = \square + \square \times G$. **2**

(e) Using your solution to part (d), estimate the number of gun deaths in a country where 1.2 million guns are held by the public, rounding your solution to the nearest whole number. **1**

Question 24

A new hybrid car was being tested in a top-secret facility. For a particular speed test, the car moved along a straight road where its velocity v m/s after time t seconds is given by the formula:

$$v = 85 + 20e^{-0.3t}.$$

Suppose that the car has an initial displacement at the starting point, i.e. $x = 0$ when $t = 0$.

(a) In terms of t, find an expression for the particle's acceleration a and hence state whether the car is speeding up or slowing down. **2**

(b) In terms of t, find an expression for the particle's displacement x. **2**

(c) Find the car's limiting velocity? **1**

Question 25

Arna wants to buy an apartment currently valued at $800,000. In doing his research, Arna concludes that the best option is to go with the *Bank of Trust* (*BoT*) who is offering interest at 5.50% p.a. (compounding monthly) where the loan will be paid over 20 years. *BoT* requires that Arna pays 20% of the purchase as a deposit and *BoT* will lend him the remaining amount. **3**

Below is a table that contains monthly repayment factors for every $1000 borrowed across different monthly periods and interest rates.

Monthly repayment factors per $'000 loan amount					
No. of Months	**Interest Rate (per period)**				
	0.396%	**0.417%**	**0.438%**	**0.458%**	**0.479%**
60	18.7569	18.8712	18.9860	19.1012	19.2168
120	10.4848	10.6066	10.7292	10.8526	10.9769
180	7.7783	7.9079	8.0388	8.1708	8.3041
240	6.4622	6.5996	6.7384	6.8789	7.0208
300	5.7012	5.8459	5.9925	6.1409	6.2911

Using the table of values, find the total amount that Arna pays for the apartment, rounding your solution to the nearest cent.

Question 26

(a) Show that $\frac{d}{dx}(x\cos x - \sin x) = -x\sin x$. **2**

(b) Hence, or otherwise, find $\int_0^{\frac{\pi}{4}} x\sin x \, dx$. **2**

Question 27

Bob and Robbin played a simple game where Bob would roll a standard die with six faces (1, 2, 3, 4, 5 and 6). **2**

The game had two rules:

- **Rule 1**: If Bob rolled a 1, 2, 3 or 4, Robbin would pay Bob the square of the amount that was shown on the die (e.g. If Bob rolled a 3 then Robbin would pay him $9).
- **Rule 2**: If Bob rolled a 5 or 6 then Bob would pay Robbin double the amount that was shown on the die (e.g. If Bob rolled a 5 then he would pay Robbin $10).

Find Bob's expected winning / losses to the nearest cent.

Question 28

At a *Grant's Green Grocer*, watermelons are usually the most popular produce sold. The watermelons available usually weigh 1.5kg each, where the weights of all the watermelon generally follow a normal distribution with standard deviation of 0.2kg. Over a busy week, 450 watermelons were sold. Using the following Standard Normal distribution table,

z	First Decimal Place									
	.0	**.1**	**.2**	**.3**	**.4**	**.5**	**.6**	**.7**	**.8**	**.9**
0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Rounding your solution to the nearest whole number,

(a) Find the number of watermelons that weighed less than or equal to 1.6kg. **2**

(b) Find the number of watermelons that weighed between 1.25kg and 1.55kg. **2**

Question 29

A continuous random variable X has a probability density function:

$$f(x) = \begin{cases} 5^x & 0 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of a, rounding to 3 decimal places. **2**

(b) Find the equation of the cumulative distribution function $F(x)$. **2**

(c) Using (b), or otherwise, find the value of the 9th decile, rounding to 3 decimal places. **1**

Question 30

At the start of 2010, Prim borrowed \$600,000 from a bank to buy an apartment where the loan was repaid with monthly instalments at an interest rate of 4.5% p.a. The loan is to be fully repaid after 30 years, i.e. by the end of 2039.

The amount owning on the loan after n months is denoted by A_n.

(a) Show that the amount owing after the n^{th} repayment is made is: **3**

$$A_n = 600000 \times 1.00375^n - \frac{800M\left(1.00375^n - 1\right)}{3}.$$

(b) Hence, or otherwise, show that the monthly repayment M is \$3040.11. **1**

(c) At the start of 2020, as a result of the onset of COVID-19, the bank agreed to allow Prim to freeze repayments where no interest would be charged. This freezing period lasted 18 months where monthly repayments resumed in mid-2021.

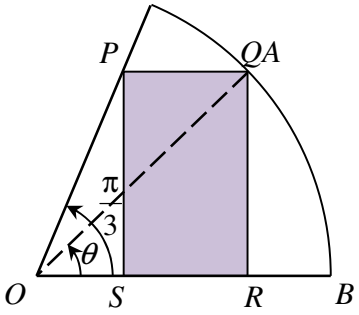
Using part (b) and rounding your solution to the nearest cent,

(i) Show that the amount owing on the loan at the start of 2020 is \$480,536.89. **1**

(ii) Using the amount in part (i), find how much Prim's monthly repayment increases by if she were to fully repay the loan according to the original end date i.e. by the end of 2039. **2**

Question 31

AOB is a sector of a circle with the centre at O, radius r units and $\angle AOB = \frac{\pi}{3}$. $PQRS$ is a rectangle drawn in the sector where $\angle AOB = \theta$, as shown in the diagram below.



(a) Show that: $SR = r\cos\theta - \frac{r\sin\theta}{\sqrt{3}}$. **2**

(b) Given that $\sin 2\theta = 2\sin\theta\cos\theta$, show that the area of rectangle $PQRS$ is given by: **2**

$$A = r^2\left(\frac{1}{2}\sin 2\theta - \frac{\sqrt{3}}{3}\sin^2\theta\right).$$

(c) Find the value of θ that will give the maximum area of $PQRS$. **3**

End of paper.