Student details

Name: _____

Mark: _____

2025

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 6 – 11

60 marks

- Attempt Questions 11 – 25
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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1 Which of the following is the solution for *x:* $\frac{2x-1}{3x+1} \leq 0$?

(A) $x \in \left(-\frac{1}{3}, \frac{1}{2}\right]$

(B) $x \in \left(-\infty, -\frac{1}{3}\right) \cap \left[\frac{1}{2}, \infty\right)$

(C) $x \in \left[-\frac{1}{2}, \frac{1}{3}\right)$

(D) $x \in \left(-\infty, -\frac{1}{2}\right] \cap \left[\frac{1}{3}, \infty\right)$

2 What is the derivative of $y = \cos^{-1}\left(x^4\right)$?

(A) $-\frac{4x^3}{\sqrt{1-x^8}}$

(B) $\frac{4x^3}{1+x^8}$

(C) $\frac{1}{\sqrt{1-x^4}}$

(D) $\frac{1}{1+x^8}$

3 What is the size of the angle between the vectors $\underset{\sim}{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$?

(A) 63°

(B) 66°

(C) 114°

(D) 117°

4 What is the coefficient of x^4 in the expansion of $(4x - 3)^9$

(A) ${}^9C_4 (4)^5 (-3)^4$

(B) ${}^9C_5 (4)^4 (-3)^5$

(C) $(4)^5 (3)^4$

(D) ${}^9C_3 (4)^6 (-3)^3$

5 What is the exact value (with surds) of $\tan\left(2\cos^{-1}\frac{a}{b}\right)$ in terms of a and b?

(A) $\dfrac{2a\sqrt{b^2 - a^2}}{2a^2 - b^2}$

(B) $\dfrac{2a}{\sqrt{b^2 - a^2}}$

(C) $\dfrac{\sqrt{b^2 - a^2}}{a^2 - 2a^2}$

(D) $\dfrac{2a - b^2}{\sqrt{b^2 - a^2}}$

6 A committee of six was to be selected from seven women and six men. How many combinations are possible if the committee is comprised of at least five women?

(A) 21

(B) 85

(C) 126

(D) 133

7 In a test with 20 multiple choice questions, each with four choices of *A*, *B*, *C* and *D*, what is the probability of a student getting a score less than 10% if the test was completed randomly?

(A) $^{20}\mathrm{C}_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{20} + {}^{20}\mathrm{C}_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^{19}$

(B) $^{20}\mathrm{C}_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{20} + {}^{20}\mathrm{C}_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^{19} + {}^{20}\mathrm{C}_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^{18}$

(C) $^{20}\mathrm{C}_{20}\left(\frac{1}{4}\right)^{20}\left(\frac{3}{4}\right)^0 + {}^{20}\mathrm{C}_{19}\left(\frac{1}{4}\right)^{19}\left(\frac{3}{4}\right)^1 + {}^{20}\mathrm{C}_{18}\left(\frac{1}{4}\right)^{18}\left(\frac{3}{4}\right)^2$

(D) $^{20}\mathrm{C}_{20}\left(\frac{1}{4}\right)^{20}\left(\frac{3}{4}\right)^0 + {}^{20}\mathrm{C}_{19}\left(\frac{1}{4}\right)^{19}\left(\frac{3}{4}\right)^1$

8 What is the value for *k* such that the function $y = \log_e\left(\frac{1}{1+x}\right)$ satisfies the differential equation $x\frac{dy}{dx} + k = e^y$?

(A) $k = e$

(B) $k = -e$

(C) $k = -1$

(D) $k = 1$

9 A polynomial $P(x)$ has a remainder of 11 when divided by $(x - 3)$ and a remainder of 4 when divided by $(x + 4)$.

What is the remainder when $P(x)$ is divided by $(x - 3)(x + 4)$?

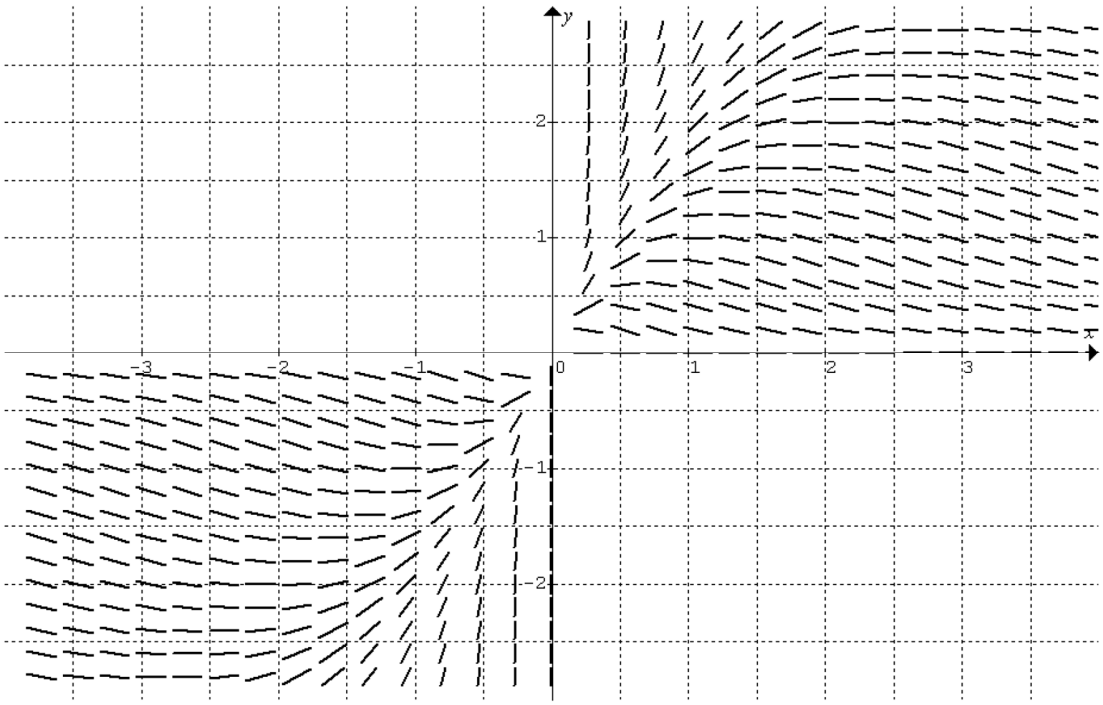
(A) 12

(B) $x + 8$

(C) $2x + 9$

(D) $3x - 5$

10 Which of the following could be the differential equation represented by the slope field shown?



(A) $\frac{dy}{dx} = x \log_e (y)$

(B) $\frac{dy}{dx} = y \log_e (x)$

(C) $\frac{dy}{dx} = \frac{y}{x} e^{\frac{y}{x}}$

(D) $\frac{dy}{dx} = \frac{y}{x} \log_e \left(\frac{y}{x} \right)$

Section II

60 marks
Attempt Questions 11–25

In Questions 11–25, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Consider the two vectors $\underset{\sim}{p} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$ and $\underset{\sim}{q} = \begin{pmatrix} -2 \\ -8 \end{pmatrix}$. **2**

Find $proj_{\underset{\sim}{p}}\underset{\sim}{q}$.

Question 12

Use the substitution $u = x^2$ to evaluate $\int_0^{\sqrt[4]{3}} \frac{x}{1 + x^4}\, dx$. **3**

Question 13

Prove by mathematical induction for integers $n \geq 1$: **3**

$$1\times3\times5 + 2\times4\times6 + \ldots + n(n+2)(n+4) = \frac{1}{4}n(n+1)(n+4)(n+5).$$

Question 14

(a) Find the area enclosed between the curve $y = \sin x + 1$ and the x-axis, between $x = 0$ and $x = \frac{\pi}{2}$. **2**

(b) Find the volume when the area in part (a) is rotated about the x-axis. **2**

Question 15

The polynomial $P(x) = x^4 + ax^3 + bx^2 + 108x - 135$ has a root of multiplicity 3 at $x = 3$.

(a) Find the value of a and b. **3**

(b) Hence, or otherwise, factorise $P(x)$ fully. **1**

Question 16

Rod spilt coffee on the floor, forming an expanding circle where the radius was expanding at a rate of 1.3 cm per second. At what rate is the surface area of the coffee increasing when the radius of the spill was 4cm? Round your solution to one decimal place. **2**

Question 17

(a) Show that $\frac{d}{dx}\left(x \tan^{-1} 3x\right) = \tan^{-1} 3x + \frac{3x}{1 + 9x^2}$. **1**

(b) Hence, evaluate $\int_0^{\frac{1}{\sqrt{3}}} \tan^{-1} 3x \, dx$. **2**

Question 18

Five girls and two boys are queuing up in a straight line to purchase tickets to a movie. How many possible arrangements are possible if:

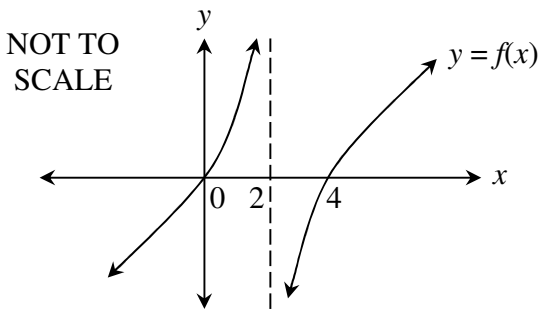
(a) No restrictions applied. **1**

(b) Both boys are next to each other in the queue. **1**

(c) There are exactly two girls separating the two boys. **2**

Question 19

The diagram shows the graph of a function $f(x)$.



Sketch the following curves on separate diagrams, showing all key features.

(a) $y = f(-x)$ **1**

(b) $|y| = f(|x|)$ **2**

(c) $y = \dfrac{1}{f(x)}$ **2**

Question 20

Consider the function $f(x) = \dfrac{x-3}{x-2}$.

(a) Find the domain of $f(x)$, stating your solution in set notation. **1**

(b) Find the inverse function $y = f^{-1}(x)$. **2**

(c) Hence, sketch the inverse function $y = f^{-1}(x)$ on a number plane, labelling all key features. **2**

Question 21

On a small island north of Hawaii, 5000 butter-nose turkeys were introduced to the existing wildlife, where its population is forecast to grow according to the logistic equation: **4**

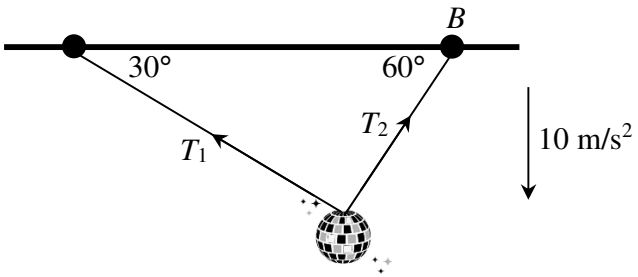
$$\frac{dP}{dt} = \frac{4P}{5}\left(1 - \frac{P}{30000}\right),$$

where P is the number of turkeys after t years.

Given that $\dfrac{30000}{P(30000 - P)} = \dfrac{1}{P} + \dfrac{1}{30000 - P}$, find the population of butter-nose turkeys three years after introduction.

Question 22

In a dance hall, a disco ball of mass 6 kilograms is currently being suspended by light inextensible ropes at two points, A and B, on the ceiling at 30° and 60° respectively. The tension between the disco ball and the two points are T_1 and T_2 respectively, as shown in the diagram below.



Assuming gravity of 10 m/s^2, and by resolving horizontal and vertical forces acting on the disco ball,

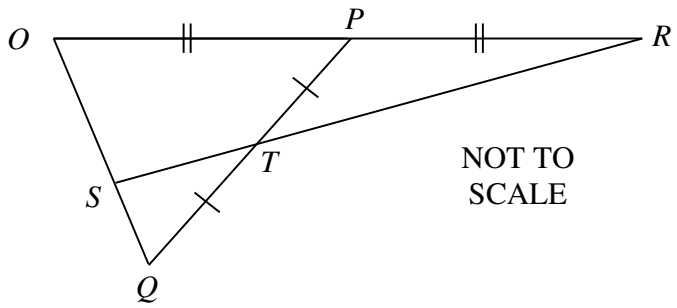
(a) Find the value of the two tensions T_1 and T_2. **2**

(b) The owner of the dance hall noticed that suspension point at A was starting to wear over time, and he was worried about the point's structural integrity. As such, he moved the suspension point at B so that it now makes a new angle with the ceiling and the tension in the rope attached to point B (T_2) has increased. By doing this, the tension in the rope attached to point A (T_1) has decreased by a third. **3**

Assuming that all ropes are in tension, find the new tension in the rope attached to point B (T_2). Round your solution to the nearest whole number.

Question 23

The side OP in ΔOPQ is produced to R such that $OP = PR$. T is the midpoint of PQ and RT produced meets OQ at S.



Let $\overrightarrow{OP} = \underset{\sim}{p}$, $\overrightarrow{OQ} = \underset{\sim}{q}$ and $\overrightarrow{RS} = \lambda\overrightarrow{RT}$ for some constant λ.

(a) Show that $\overrightarrow{OS} = \left(2 - \frac{3}{2}\lambda\right)\underset{\sim}{p} + \frac{1}{2}\lambda\underset{\sim}{q}$. **3**

(b) By letting $\overrightarrow{OS} = \mu\overrightarrow{OQ}$ for some constant μ, show that $OS : SQ = 2 : 1$. **3**

Question 24

A, B and C are the vertices of a triangle. **4**

Given $\sin X + \sin Y = 2\sin\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$

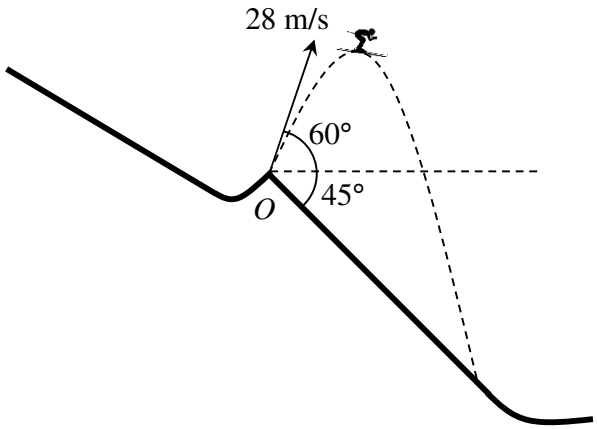
and $\cos X + \cos Y = 2\cos\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$ **[DO NOT PROVE THIS]**.

Prove the identity: $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$.

Question 25

One of the key highlights of the 3026 Winter Olympic games was when Venna Von Vinterburg won gold in the Ski Jumping event, just ousting her main rival Chi Chi Yen. In the jump event, the skier accelerates down a slope and then launches off a jump ramp and then lands on a straight slope inclined at 45°. The skier that landed the furthest from the jump ramp is the winner.

For Venna's gold medal jump, she was able to attain a take-off velocity of 28 m/s, launching at an angle of 60° to the horizontal, as shown in the diagram below.



Let the origin O be the launch point on the jump ramp.

The displacement vector of Venna's jump is $\underset{\sim}{s} = (14t)\underset{\sim}{i} + \left(-5t^2 + 14\sqrt{3}t\right)\underset{\sim}{j}$.

(a) Find the velocity vector $\underset{\sim}{v}$ of Venna's jump. **1**

(b) Show that the Cartesian equation of Venna's motion is: **1**

$$y = -\frac{5x^2}{196} + \sqrt{3}x$$

(c) Show that the time (T) that Venna is in the air between the jump ramp and landing on the straight slope is: **2**

$$T = \frac{14\left(\sqrt{3} + 1\right)}{5} \text{ seconds.}$$

(d) Hence, or otherwise, show that Venna's landing velocity (V) of the straight slope is: **2**

$$V = 28\sqrt{\sqrt{3} + 2} \text{ m/s.}$$

End of paper.