NExT EDUCATION

2019

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used

Total marks – 70

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 7 – 13

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

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1 Which of the following is equivalent to $\int \frac{3}{\sqrt{4 - x^2}}\, dx$?

(A) $\frac{3}{2}\sin^{-1}\left(\frac{x}{2}\right) + c$

(B) $3\sin^{-1}\left(\frac{x}{2}\right) + c$

(C) $\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

(D) $3\cos^{-1}\left(\frac{x}{2}\right) + c$

2 What is the solution to x: $\frac{4}{x-3} \le 2$

(A) $3 \le x \le 5$

(B) $3 < x \le 3$

(C) $x < 3$ or $x \ge 5$

(D) $x \le 3$ or $x \ge 5$

3 The polynomial $P(x) = x^3 - 8x^2 + 3x - 15$ has roots α, β and γ.

What is the value of $\alpha^2 + \beta^2 + \gamma^2$?

(A) -7

(B) 58

(C) 73

(D) 219

4 What is the value of $\lim\limits_{x \to \infty} \dfrac{6x^4 - 5x^3 + 2x^2 - 1}{8x^2 + 2x^4}$?

(A) 0

(B) 0.75

(C) 3

(D) ∞

5 If $\sqrt{3}\cos x - \sin x = R\cos(x + \alpha)$, which of the following gives the correct value of α?

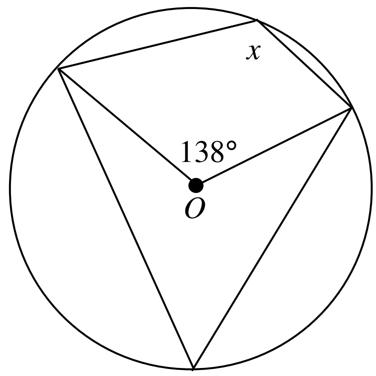
(A) $\dfrac{\pi}{6}$

(B) $\dfrac{5\pi}{6}$

(C) $\dfrac{\pi}{3}$

(D) $\dfrac{\pi}{2}$

6 In the following diagram, O is the centre of the circle.



What is the value of x?

(A) 111°

(B) 42°

(C) 152°

(D) 69°

7 What are the coordinates of the point that divides the interval AB externally into the ratio 5 : –3, given that the coordinates of A and B are (9,4) and (3,2) respectively?

(A) (–6,–1)

(B) (–6,1)

(C) (6,–1)

(D) (6,1)

8 What is the domain and range of the inverse function $y = 3\cos^{-1}\dfrac{x}{4}$?

(A) Domain: $0 \le x \le 4$; Range: $-3\pi \le y \le 3\pi$

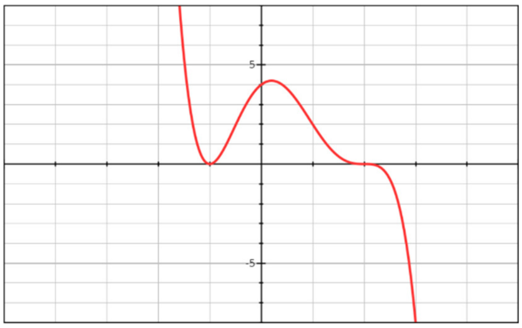
(B) Domain: $-3 \le x \le 3$; Range: $-4\pi \le y \le 4\pi$

(C) Domain: $-4 \le x \le 4$; Range: $0 \le y \le 3\pi$

(D) Domain: $-\dfrac{1}{3} \le x \le \dfrac{1}{3}$; Range: $-2\pi \le y \le 2\pi$

9 Which of the following graphs best represents the function $y = 2x(2-x)^2(1-x^2)$?

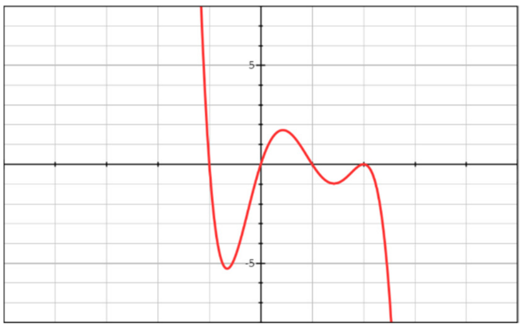
(A)



(B)

(C)

(D)



10 Sam and Gilly play a series of games, where the first person to win two games in a row wins the series. For each game in the series, Sam has a probability of 0.4 to win and Gilly has a probability of 0.6 to win.

Assuming that they play until an eventual winner is declared, what is the probability that Sam wins?

(A) 0.21

(B) 0.4

(C) 0.53

(D) 0.68

End of Section I.

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question on a NEW page on your OWN PAPER.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW page on your OWN PAPER.

(a) Factorise fully $2x^3 - 128y^3$. **2**

(b) Find $\int \sin^2 4x \, dx$. **2**

(c) Use the substitution $u = x - 1$ to evaluate $\int_2^4 \frac{x}{(x-1)^2} \, dx$. **3**

(d) Differentiate $y = xe^{\sin x}$. **2**

(e) If $\alpha = \tan^{-1} \frac{5}{12}$ and $\beta = \cos^{-1} \frac{4}{5}$, calculate the exact value of $\tan(\alpha - \beta)$. **2**

(f) (i) Show that the equation $e^x = x + 2$ has a solution in the interval $1 < x < 2$. **2**

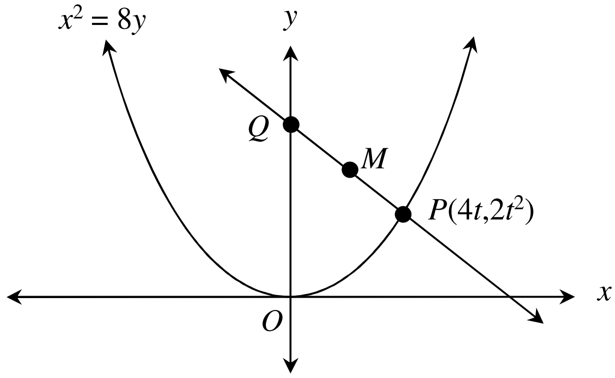
(ii) By letting $x_0 = 1.5$, use one application of Newton's Method to approximate the solution, round to three decimal places. **2**

End of Question 11.

Question 12 (15 marks) Use a NEW page on your OWN PAPER.

(a) Find the term independent of x in the expansion of $\left(3x^3+2\right)\left(4x^2-\frac{3}{x}\right)^6$. **4**

(b) The point $P(4t, 2t^2)$ lies on the parabola $x^2 = 8y$. The normal at P cuts the y-axis at the point Q, where the midpoint of PQ is M, as shown in the diagram.



(i) Show that the equation of the normal at P is $x + ty = 4t + 2t^3$. **1**

(ii) Find the coordinates of Q. **1**

(iii) Find the equation of the locus of M as P moves on the parabola. **2**

(c) A particle is oscillating in simple harmonic motion such that its displacement x metres from the origin O after t seconds satisfies the equation:

$$\ddot{x} = -4x\,.$$

(i) Show that $x = \alpha\cos\left(2t+\beta\right)$ is a possible equation of motion for the particle, where α and β are constants. **1**

(ii) Initially, the particle has a displacement of 4m and a velocity of 8 m/s. Find the amplitude of the oscillation. **2**

Question 12 continues on the next page.

(d) (i) Prove using principles of mathematical induction for all integers $n \geq 1$ that: **3**

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1).$$

(ii) Hence, show that $2^2 + 4^2 + 6^2 + \ldots + 100^2 = 171700$. **1**

(iii) Using the results of the above, find the value of $1^2 + 3^2 + 5^2 + \ldots + 99^2$. **1**

End of Question 12.

Question 13 (15 marks) Use a NEW page on your OWN PAPER.

(a) Newton's Law of Cooling states that the rate of cooling of a body is proportional to the excess of the temperature of a body above the surrounding temperature. This rate can be represented by the differential equation:

$$\frac{dT}{dt} = -k(T - R)$$

where T is the temperature of the body, R is the temperature of the surroundings, t is the time in minutes and k is a constant.

(i) Show that $T = R + Ae^{-kt}$ is a solution to the differential equation, where A is a constant. **1**

(ii) A glass of milk cools from 85°C to 80°C in one minute in a room of constant temperature of 25°C. Find the temperature, to the nearest degree, of the glass of milk after a <u>further four minutes</u> have elapsed. **2**

(b) In the expansion of $(1+x)^n$, the coefficients of x, x^2 and x^3 form an arithmetic progression.

(i) Show that $2\binom{n}{2} = \binom{n}{1} + \binom{n}{3}$. **1**

(ii) Hence, show that $n^3 - 9n^2 + 14n = 0$. **2**

(iii) Hence, find the value of n that satisfies the above condition to form an arithmetic progression. **2**

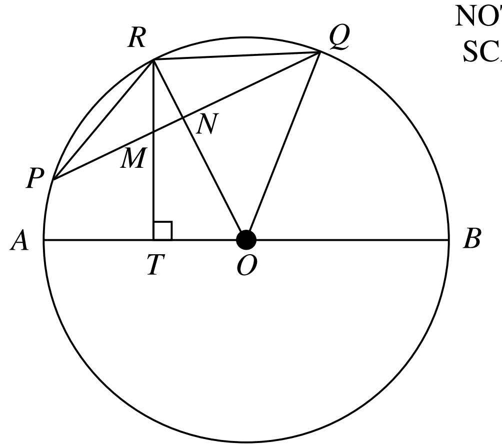
(c) From the letters of the word 'G L A S S E S',

(i) How many arrangements of all the letters if no restrictions applied? **1**

(ii) How many arrangements are possible if only five letters are chosen? **3**

Question 13 continues on the next page.

(d) In the following diagram, AB is a diameter of the circle with centre O. PQ is a chord of the circle with R being a point on the circumference such that $PR = RQ$. RT is a perpendicular drawn from R to AB. **3**



NOT TO SCALE

Prove that $TMNO$ is a cyclic quadrilateral.

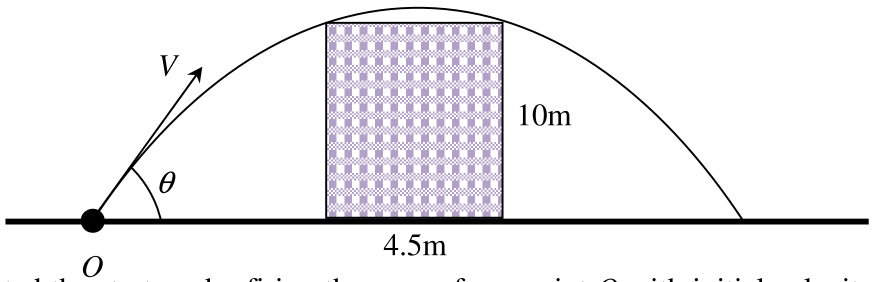
End of Question 13.

Question 14 (15 marks) Use a NEW page on your OWN PAPER.

(a) (i) Show that: $\frac{d}{dx}\left(x\sqrt{1-x^2}+\sin^{-1}x\right)=2\sqrt{1-x^2}$. **2**

(ii) Hence, evaluate $\int_0^{\frac{1}{2}}\sqrt{1-x^2}\,dx$ leaving your solution in exact form. **2**

(b) In the TV show '*Game of Kings*', *Ron Snow* was a brave leader who was about to storm walls of *Queen's Landing*. As part of his strategy to storm the wall, he needed to fire an arrow tied to a rope up and over the wall, as shown in the diagram below.



Ron executed the strategy by firing the arrow from point O with initial velocity of V m/s at an angle of elevation θ, which just cleared both sides of the wall and landed on the ground. The wall was 10 metres tall and 4.5 metres wide.

After t seconds, the horizontal (x) and vertical (y) displacements of the arrow are given as follows: $x = Vt\cos\theta$ and $y = -\frac{gt^2}{2} + Vt\sin\theta$ (**DO NOT PROVE THESE**) where gravity is g m/s^2.

(i) Show that the horizontal range R of the arrow is $\frac{V^2\sin 2\theta}{g}$. **2**

(ii) Hence, show that the equation of the path of the arrow is $y = x\left(1-\frac{x}{R}\right)\tan\theta$. **2**

(iii) If Ron fired the arrow at an angle of 45°, show that the horizontal distances (x-coordinates) of the wall from point O are the roots of the equation: **1**

$$x^2 - Rx + 10R = 0.$$

(iv) Hence, find the value of R. **2**

Question 14 continues on the next page.

(c) (i) Given $(1+x)^p + (1+x)^{p+1} + \ldots + (1+x)^{p+q} = \dfrac{(1+x)^{p+q+1} - (1+x)^p}{x}$ **2**

where p and q are positive integers and $x \neq 0$. (**DO NOT PROVE THIS**).

Show that: ${}^{p}\text{C}_{p} + {}^{p+1}\text{C}_{p} + \ldots + {}^{p+q}\text{C}_{p} = {}^{p+q+1}\text{C}_{p+1}$.

(ii) Hence, or otherwise, show that: **2**

$$\sum_{r=5}^{q+4} r(r-1)(r-2)(r-3) = 24\left({}^{q+5}\text{C}_5 - 1\right).$$

End of paper.