Student details

Name: _____

Mark: _____

2023

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 6 – 12

60 marks

- Attempt Questions 11 – 30
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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1 Which of the following is the solution to x in the equation $(x - 3)(2 - x)(1 + x) = 0$?

(A) $x = 3, 2, -1$

(B) $x = 3, -2, 1$

(C) $x = -3, -2, 1$

(D) $x = 3, -2, -1$

2 Which of the following is equivalent to the magnitude of the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$?

(A) 5

(B) $\sqrt{5}$

(C) $2\sqrt{5}$

(D) 25

3 Which of the following is equal to $\sqrt{3}\cos x - \sin x$?

(A) $2\cos\left(x + \frac{\pi}{6}\right)$

(B) $2\cos\left(x - \frac{\pi}{3}\right)$

(C) $2\cos\left(x + \frac{\pi}{3}\right)$

(D) $2\cos\left(x - \frac{\pi}{6}\right)$

4 If $(x - 3)$ is a factor of $P(x) = x^3 - 7x^2 + ax + 6$, what is the value of a?

(A) 6

(B) 10

(C) 15

(D) 21

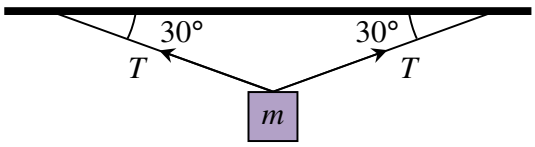
5 A disco ball weighing m kg was suspended in air held by two light inextensible strings attached to a horizontal ceiling. Both strings make angles of 30° with the ceiling, as shown in the diagram below. Assuming gravity of g ms^{-2}, which of the following is equivalent to the tension T in both strings?

(A) mg

(B) $2\sqrt{3}mg$

(C) $\frac{\sqrt{3}}{2}mg$

(D) $2mg$



6 Which of the following is equivalent to $\dfrac{\cot\frac{x}{2} - \tan\frac{x}{2}}{\cot\frac{x}{2} + \tan\frac{x}{2}}$?

(A) $\cos x$

(B) $\sec x$

(C) $\tan x$

(D) $\cot x$

7 A game was played where positive integers were written on individual cards, the cards were then randomly selected and the integer value was divided by 7. The remainder, which could be 0, 1, 2, 3, 4, 5 or 6, was observed and recorded.

What is the smallest number of cards such that all remainders have at least one of the remainder values has 9 observations?

(A) 56

(B) 57

(C) 63

(D) 64

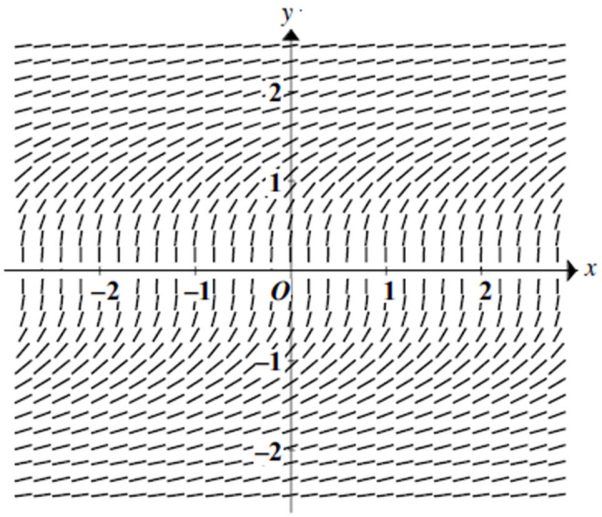
8 Which of the following could be the differential equation represented by the slope field below?

(A) $\frac{dy}{dx} = \frac{1}{\sqrt{4 - x^2}}$

(B) $\frac{dy}{dx} = \frac{2}{1 + x^2}$

(C) $\frac{dy}{dx} = \frac{1}{y^2}$

(D) $\frac{dy}{dx} = \cos(y)$



9 Let n be a positive integer. Which of the following equals to the coefficient of a^3b^5 in the expansion of $\left(1 + ab + b^2\right)^n$?

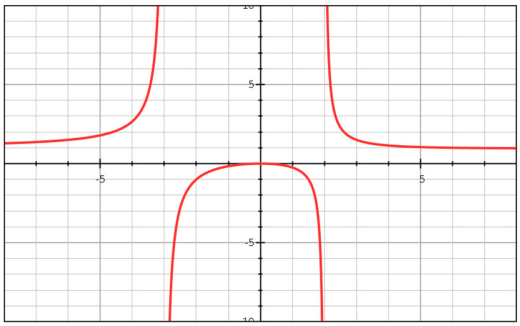
(A) ${}^n\mathrm{C}_2$

(B) 2^n

(C) ${}^n\mathrm{C}_3 \, {}^n\mathrm{C}_5$

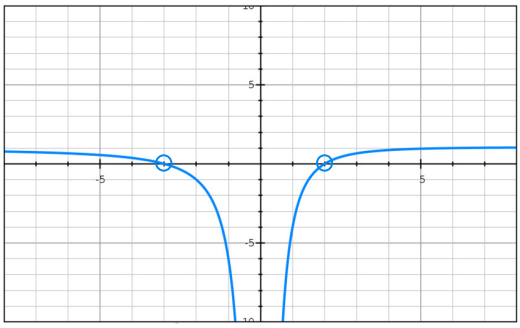
(D) $4 \, {}^n\mathrm{C}_4$

10 The following is a sketch of the function $f(x) = \dfrac{x^2}{(x+3)(x-2)}$.

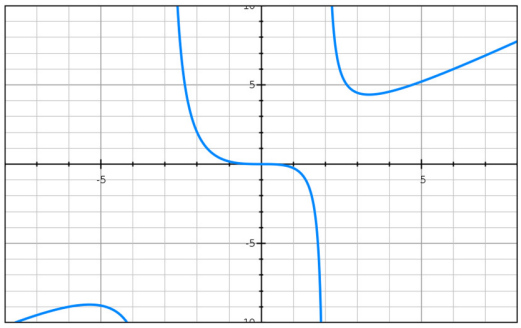


Which of the following **DOES NOT** represent one of the transformations $\sqrt{f(x)}$, $\dfrac{1}{f(x)}$ or $xf(x)$?

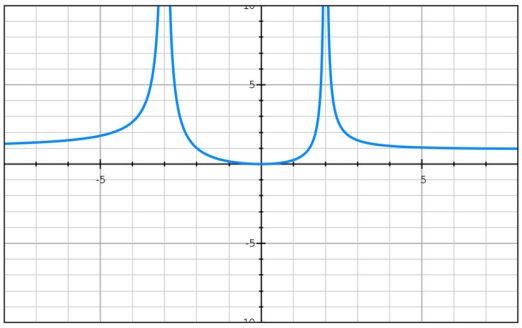
(A)



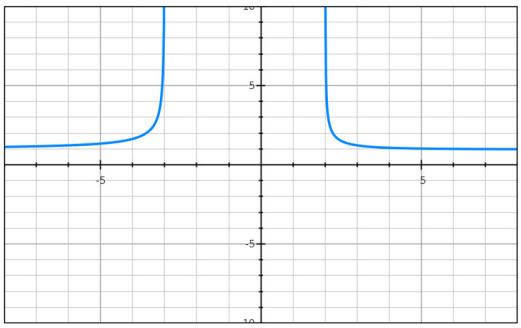
(B)



(C)



(D)



Section II

60 marks
Attempt Questions 11–30

In Questions 11–30, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Find $\int \frac{1}{4+x^2}\,dx$. **1**

Question 12

Solve for x, expressing you solution in set notation: $\frac{2x-1}{x+3} \leq 1$. **2**

Question 13

Consider the two vectors $\underset{\sim}{a} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

(a) Find the size of the acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$, rounding your solution to the nearest degree. **1**

(b) Find the unit vector of $\underset{\sim}{a}$. **1**

(c) Find $proj_{\underset{\sim}{a}}\underset{\sim}{b}$. **1**

Question 14

Find the exact value of $\cos\left(\sin^{-1}\frac{3}{7}\right)$. **2**

Question 15

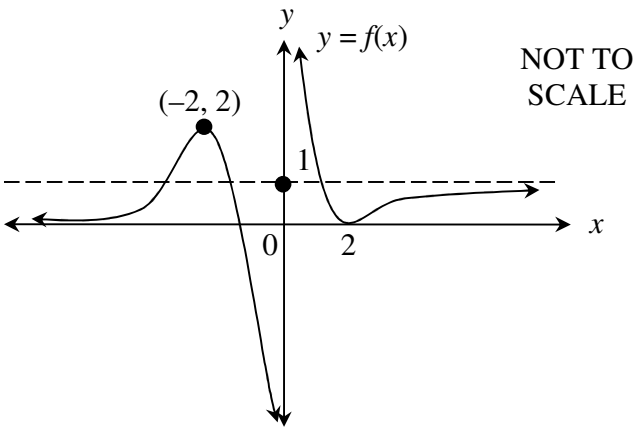
Prove the following identity: $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$. **2**

Question 16

Seven friends, Ant, Bec, Cat, Dat, Edd, Fen and Gen, sat around a circular table. How many arrangements are possible if Ant, Bec and Cat sat next to each other in a group, and Dat, Edd and Fen sat next to each other in a group? **2**

Question 17

The diagram shows the graph of a function $f(x)$.



Sketch the following curves on separate diagrams:

(a) $y = -f(x)$ **1**

(b) $y = f\left(|x|\right)$ **1**

(c) $y^2 = f(x)$ **2**

(d) $y = \dfrac{1}{f\left(x\right)}$ **2**

Question 18

Prove the identity ${}^nC_{n-k} = {}^nC_k$. **2**

Question 19

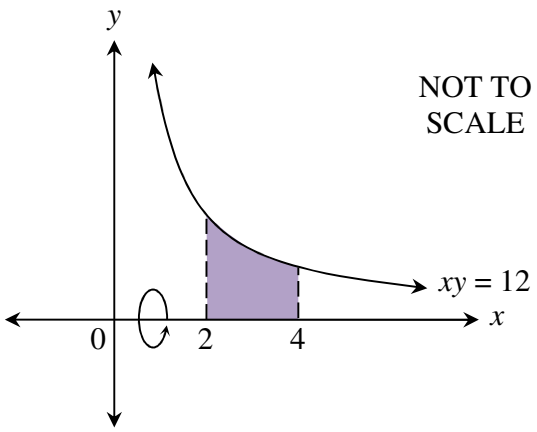
Find the solution to the differential equation: $\frac{dy}{dx} - \frac{2ye^{2x}}{1+e^{2x}} = 0$, given $y = 1$ when $x = 0$. **2**

Question 20

Use the substitution $x = \sin\theta$ to evaluate $\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx$. **3**

Question 21

The area enclosed by the curve $xy = 12$, the x-axis and the lines $x = 2$ and $x = 4$ is rotated about the x-axis, as shown in the diagram below. **2**



Find the volume formed by the rotation.

Question 22

The polynomial $P(x) = x^3 + x^2 - x - 7$ has roots α, β and γ.

(a) Find the value of $\alpha + \beta + \gamma$. **1**

(b) Given that $\alpha^2 + \beta^2 + \gamma^2 = 3$ and $\alpha^3 + \beta^3 + \gamma^3 = 17$, find the value of $\alpha^4 + \beta^4 + \gamma^4$. **2**

Question 23

Consider a rectangle with dimensions of 6cm by 10cm. **2**

By using the Pigeonhole Principle, show that if sixteen points were drawn within the rectangle then at least two points will be less than $2\sqrt{2}$ cm apart.

Question 24

Find the coefficient of x^5 in the expansion of $\left(3x^2 - \frac{4}{x}\right)^{12}\left(\frac{2}{x} - x^2\right)$. **3**

Question 25

Prove by mathematical induction for $n \in \mathbb{Z}^+$ that $5^{3n} + 5^{2n} + 5^n + 1$ is divisible by 4. **3**

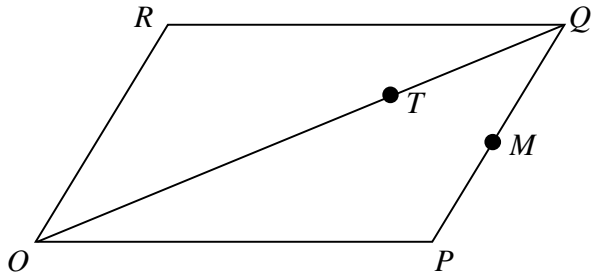
Question 26

By considering the Sums to Products properties, solve for x for $x \in [0, 2\pi]$: **3**

$$\sin 4x - \sin 2x = 0 .$$

Question 27

$OPQR$ is a parallelogram such that $\overrightarrow{OP} = \underset{\sim}{p}$ and $\overrightarrow{OR} = \underset{\sim}{r}$. The point M is the midpoint of PQ and the point T lies on OQ such that $OT : TQ = 2 : 1$, as shown on the diagram below.



NOT TO SCALE

(a) In terms of $\underset{\sim}{p}$ and $\underset{\sim}{r}$, find the expressions for the vectors $\overrightarrow{RT}$ and $\overrightarrow{RM}$. **2**

(b) Hence, or otherwise, show that the R, T and M are collinear. **1**

Question 28

Sven was about to sit a test that comprised 25 multiple choice questions, each with four choices of A, B, C or D. As an advocate of fate and chance, he decided to answer all questions randomly. For this test, Sven's potential outcome can be considered to follow a binomial distribution since the potential outcome for each question can either be correct or incorrect.

(a) Find the probability that Sven gets less than 3 questions correct, rounding your solution to three significant figures. **2**

(b) Find the number of questions Sven is expected to get correct. **1**

(c) Assume the Normal approximation is appropriate for this Binomial distribution. Using the Standard Normal table of values below, estimate the probability that Sven gets less than 5 questions correct, rounding your solution to three significant figures. Ignore the continuity correction adjustment. **2**

z	First Decimal Place									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.0	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.0	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.0	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Question 29

Zin, a scientist, has recently made a ground-breaking discovery. He found that when he combined the rare chemical *Xonium* with a small amount of *Yitrix*, a new chemical is formed – in the lab he has been calling the new chemical *Zinium* (after himself). In a controlled experiment Zin combined five grams of *Xonium* with three grams of *Yitrix* and noticed that the two substances combined in equal parts (in terms of mass) to form *Zinium*.

According to his notes the velocity of reaction to produce *Zinium* is proportional to the product of the unused mass of *Xonium* and *Yitrix*. In other words, i.e. the rate of change in the mass of *Zinium* (m) after t hours is given by the differential equation:

$$\frac{dm}{dt} = k(m-6)(m-10) \ . \ \textbf{[DO NOT PROVE THIS].}$$

(a) Show that: $\dfrac{1}{(m-6)(m-10)} = \dfrac{1}{4}\left(\dfrac{1}{m-10} - \dfrac{1}{m-6}\right)$. **1**

Assuming that no amount of *Zinium* was initially present in the experiment,

(b) Show that the solution to the differential equation is: **3**

$$m = \frac{30\left(1-e^{4kt}\right)}{3-5e^{4kt}}, \text{ where } t \geq 0.$$

(c) Zin found that after four hours, two grams of *Zinium* is produced. Find the value of k, rounding your solution to three significant figures. **1**

(d) Using (c), estimate the mass of *Zinium* produced after 8 hours. Round your solution to the nearest one decimal place. **1**

Question 30

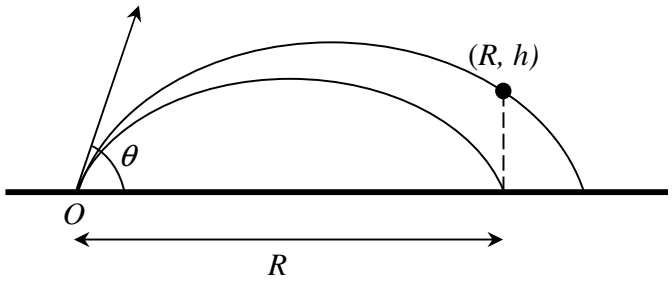
A projectile was projected from a point O on the ground with initial velocity of u m/s at an angle of θ to the horizontal. Assuming gravity is g m/s^2, the equation of the Cartesian equation of the path that the projectile travels along is given as:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \quad \textbf{(DO NOT PROVE THIS)}.$$

The horizontal range that the projectile travels along will change depending on the angle of projection, where R is the maximum possible range.

(a) Show that $R = \dfrac{u^2}{g}$. **2**

The diagram below shows two paths, one where the projectile achieves the maximum range R while the other is where the initial velocity is increased to v m/s such that it passes h metres above R.



(b) Show that $v = \dfrac{u^2}{\sqrt{u^2 - gh}}$. **3**

End of paper.