NEX

Student details

Name: ____________________

Mark: ____________________

2025

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen.
- Board-approved calculators may be used.
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 6 – 12

90 marks

- Attempt Questions 11 – 31
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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1 Which of the following is the modulus–argument (polar) form of $z = -3 + 3i$?

(A) $z = 3\sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right)$

(B) $z = 3\text{cis}\left(\frac{\pi}{3}\right)$

(C) $z = 2\sqrt{3}\text{cis}\left(-\frac{\pi}{4}\right)$

(D) $z = 6\text{cis}\left(\frac{\pi}{2}\right)$

2 What is the centre and radius of the sphere: $x^2 - 6x + y^2 + 4y + z^2 - 8z - 92 = 0$?

(A) Centre = (3, –2, 4); Radius = 11 units

(B) Centre = (6, –4, 8); Radius = $\sqrt{92}$ units

(C) Centre = (–3, 2, –4); Radius = 121 units

(D) Centre = (–6, 4, –8); Radius = 92 units

3 What is the equivalent to $\int \frac{1}{x \ln x}\, dx$?

(A) $\frac{1}{\left(\ln|x|\right)^2} + c$

(B) $x\ln|x| + c$

(C) $xe^x + c$

(D) $\ln\left|\ln|x|\right| + c$

4 Which of the following equals to the magnitude of the vector $\begin{pmatrix} \cos 2\theta \\ -\tan 2\theta \\ -\sin 2\theta \end{pmatrix}$?

(A) $\sec 2\theta$

(B) $\operatorname{cosec} 2\theta$

(C) $\cot 2\theta$

(D) $\sin 2\theta \cos 2\theta$

5 Consider the statement:

"If I had watched what I ate and slept more, then I would have been healthier and lived longer".

Which of the following represents the <u>converse</u> of the statement?

(A) *"If I had watched what I ate and slept more, then I would <u>not</u> have been healthier and would <u>not</u> have lived longer"*

(B) *"If I had watched what I ate <u>or</u> slept more, then I would have been healthier <u>or</u> lived longer"*

(C) *"If were healthier and lived longer, then I would have watched what I ate and slept more"*

(D) *"If were healthier or lived longer, then I would have watched what I ate or slept more"*

6 By considering the Euler (exponential) form, what is the exact value of i^i ?

(A) e^{π}

(B) $e^{\frac{\pi}{2}}$

(C) $e^{-\pi}$

(D) $e^{-\frac{\pi}{2}}$

7 The velocity v (in metres per second) of a particle after t seconds is given by: $v = 4x - 4$, where x is the displacement (in metres) after t seconds. Initially, the particle is 2m to the right of the origin.

Which of the following is an equation that represents the particle's displacement over time?

(A) $x = 2t^2 - 4t$

(B) $x = \log_e(t - 2)$

(C) $x = \dfrac{4}{t - 1}$

(D) $x = e^{4t} + 1$

8 Given that $|z| = 1$, where is the largest possible value for $\arg(z + 2)$?

(A) $\dfrac{2\pi}{3}$

(B) $\dfrac{3\pi}{4}$

(C) $\dfrac{\pi}{3}$

(D) $\dfrac{\pi}{6}$

9 By considering the five fifth roots of the equation $z^5 + 32 = 0$, which of the following expressions are correct?

(A) $z^4 - 2z^3 + 4z^2 - 8z + 16 = \left(z^2 - 4\cos\frac{\pi}{5}z + 4\right)\left(z^2 - 4\cos\frac{3\pi}{5}z + 4\right).$

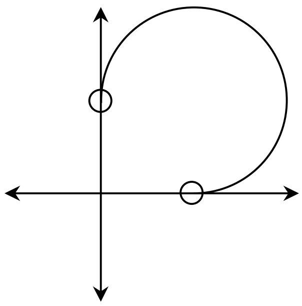
(B) $z^4 + 8z^3 + 12z^2 + 8z + 16 = \left(z^2 + 2\cos\frac{2\pi}{5}z + 4\right)\left(z^2 + 2\cos\frac{4\pi}{5}z + 4\right).$

(C) $z^4 - z^3 + z^2 - z + 16 = \left(z^2 - \cos\frac{\pi}{10}z + 4\right)\left(z^2 - \cos\frac{3\pi}{10}z + 4\right).$

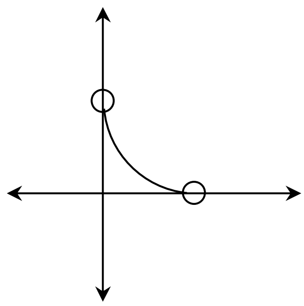
(D) $z^4 + 4z^3 + 8z^2 + 12z + 16 = \left(z^2 + 8\cos\frac{2\pi}{5}z + 4\right)\left(z^2 + 8\cos\frac{4\pi}{5}z + 4\right).$

10 Which of the following diagrams can represent $\arg\left(\frac{z-3}{z-3i}\right) = \pi$?

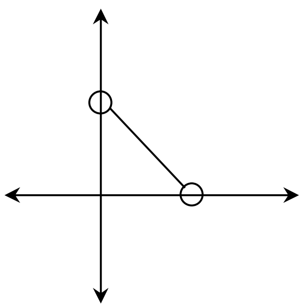
(A)



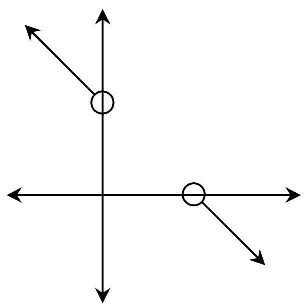
(B)



(C)



(D)



Section II

90 marks
Attempt Questions 11–31

In Questions 11–31, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

If $z = -24 - 70i$ and $w = 3 + 3i$, express each of the following in the form $a + ib$, where $a, b \in \mathbb{R}$.

(a) $\dfrac{6}{w}$. **1**

(b) $2w - \overline{z}$. **1**

(c) $\sqrt{z}$. **2**

(d) w^6. **2**

Question 12

Find $\int \sin^3 x \cos^4 x \, dx$. **2**

Question 13

If $p > 0$, $q > 0$ and $r > 0$,

(a) Show that: $p^2 + (qr)^2 \geq 2pqr$. **1**

(b) Show that: $p^2 + q^2 + r^2 \geq pq + pr + qr$. **2**

(c) Show that: $p^2(1 + q^2) + q^2(1 + r^2) + r^2(1 + p^2) \geq 6pqr$. **2**

(d) Hence, or otherwise, show that: $p^2(1 + p^2) + q^2(1 + q^2) + r^2(1 + r^2) \geq 6pqr$. **2**

Question 14

A particle moves according to the formula:

$$v^2 = 18 - 16x - 2x^2$$

where v is the velocity (in metres per second) and x (in metres) is the displacement of the particle after t seconds.

(a) Show that the particle moves with simple harmonic motion. **1**

(b) Find the period and amplitude of the particle's movement. **2**

Question 15

Consider the following vector equations of two lines:

$$\underset{\sim}{r}_1 = \begin{pmatrix} -1 \\ 2 \\ -8 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} \text{ and } \underset{\sim}{r}_2 = \begin{pmatrix} 1 \\ 5 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}.$$

(a) Find the angle between the two lines, rounding your solution to the nearest degree. **1**

(b) Find the point of intersection between the two lines. **2**

(c) Find a vector that is perpendicular to both $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ **2**

Question 16

Prove by contradiction that $\sqrt[3]{m}$ is irrational, where m is a prime number. **3**

Question 17

On an Argand diagram, shade the region where: **3**

$$\left|\frac{z+3}{z-7}\right| < 1 \quad \text{and} \quad \text{Im}\left(2z^2\right) \le 8.$$

Question 18

Find $\displaystyle\int \frac{18-6x^2}{x\left(x^2+9\right)}\,dx$. **3**

Question 19

Consider the statement: $\forall a,\, b \in \mathbb{R}^+$, If $b^3 - a^2b \le b^2 + ab$ then $b \le a+1$. **2**

By considering the contrapositive, prove the statement.

Question 20

Consider the points $A(-3,5,1)$, $B(2,7,-4)$ and $C(1,-3,-2)$.

(a) Find $proj_{\overrightarrow{AB}}\overrightarrow{AC}$. **2**

(b) Hence, or otherwise, find the area of ΔABC. **2**

Question 21

Use integration by parts to find $\displaystyle\int \log_e \sqrt{x-5}\,dx$. **3**

Question 22

Consider the following complex expression: $\left|z-3\sqrt{3}-3i\right|=3$.

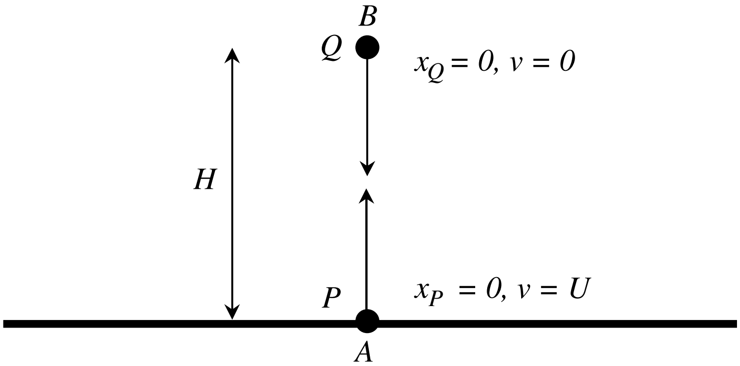
(a) Find the maximum value of $|z|$. **1**

(b) Find the range of values of $\arg(z)$. **2**

Question 23

An object P with mass m kg was projected vertically upwards from point A on the ground with initial velocity U metres per second. At the same instant, a second object Q also with mass m kg was released from rest from point B vertically above point A leading to a collision with object P. The distance H between A and B is equal to the maximum height that P would have travelled were it not to collide with Q. The objects experience air resistance of magnitude mkv^2, where k is a constant and v is velocity. Particle Q reaches 50% of its terminal velocity V at its point of collision with object P.

Let x_P be the distance of P above A and x_Q be the distance of Q below B.



Assume gravity is g m/s^2.

(a) Show that $V = \sqrt{\dfrac{g}{k}}$. **1**

(b) Show that $x_P = \dfrac{1}{2k}\log_e\left(\dfrac{g + kU^2}{g + kv^2}\right)$. **2**

(c) Hence, show that $H = \dfrac{1}{2k}\log_e\left(1 + \dfrac{U^2}{V^2}\right)$. **1**

(d) Given that $x_Q = \dfrac{1}{2k}\log_e\left(\dfrac{g}{g - kv^2}\right)$ **[DO NOT PROVE THIS]**. **1**

Show that at point of collision, $x_Q = \dfrac{1}{2k}\log_e\dfrac{4}{3}$.

(e) Show that the velocity of P at the point of collision is $\dfrac{V}{\sqrt{3}}$. **2**

Question 24

Solve for θ, where $-\pi \le \theta \le \pi$: $\left|e^{3i\theta} - i\right| = 1.$ **3**

Question 25

(a) Using the substitution $t = \tan\frac{x}{2}$, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x+\sin x}\,dx$. **2**

(b) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \frac{x}{1+\cos x+\sin x}\,dx$. **3**

Question 26

A 5kg object was projected through the air with an initial velocity of 72 m/s at an angle of 60° to the horizontal ground. In addition to gravity of 10 m/s^2, the object experiences air resistance proportional to the object's velocity v of $\frac{v}{20}$.

(a) Show that the vertical equation of motion for acceleration is $\ddot{y} = -10 - \frac{\dot{y}}{100}$. **1**

(b) Find the maximum height attained by the object, rounding your solution to the nearest metre. **3**

Question 27

Find $\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\,dx$. **3**

Question 28

(a) By considering the expansion of $\left(\cos\theta + i\sin\theta\right)^7$, show that: **2**

$$\tan 7\theta = \frac{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}.$$

(b) Using part (a), solve the equation: **2**

$$x^6 - 21x^4 + 35x^2 - 7 = 0.$$

(c) Hence, or otherwise, show that: **3**

$$\cot^2\frac{\pi}{7} + \cot^2\frac{2\pi}{7} + \cot^2\frac{3\pi}{7} = 5.$$

Question 29

Let $I_n = \int_0^{\frac{\pi}{4}} \sec^n x \, dx$ for integers $n \geq 0$.

(a) Show that $I_n = \dfrac{\left(\sqrt{2}\right)^{n-2}}{n-1} + \dfrac{n-2}{n-1} I_{n-2}$ for integers $n \geq 2$. **3**

(b) Hence, find the value of $\int_0^2 \left(4 + x^2\right)^{\frac{5}{2}} dx$. **4**

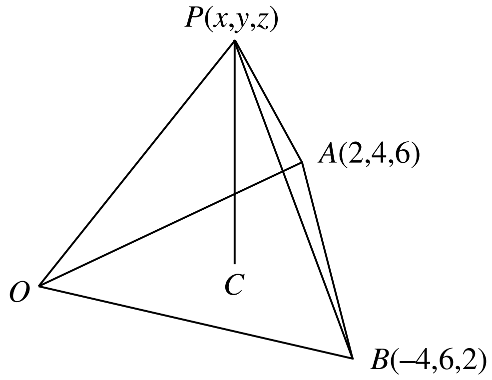
Question 30

Prove by mathematical induction for $n \in \mathbb{Z}^+$: **4**

$$\frac{1}{2} + \cos x + \cos 2x + \cos 3x + \ldots + \cos\left(nx\right) = \frac{\sin\left(n + \frac{1}{2}\right)x}{2\sin\frac{x}{2}}$$

Question 31

OABP is triangular pyramid, where *O* is the origin and coordinates of the other vertices are $A(2,4,6)$, $B(-4,6,2)$ and $P(x,y,z)$. The height of the pyramid is the length *CP*, where *C* is a point on the base *OAB* such that *CP* is perpendicular to the base.



(a) Using vectors, show that ΔOAB is an equilateral triangle. **1**

(b) *M* is the midpoint of *AB*. Given that $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OM}$, find an expression for $\overrightarrow{CP}$ in terms of *x*, *y* and *z*. **2**

(c) It is given that $OP = BP = AP = 2\sqrt{14}$. **3**

By considering similar relationships for $\overrightarrow{AC}$ and $\overrightarrow{BC}$, find the coordinates of *P*.

End of paper.