NEXT EDUCATION

2016

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 11

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

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1 What is the solution to the inequality $4x^2 \leq 12x$?

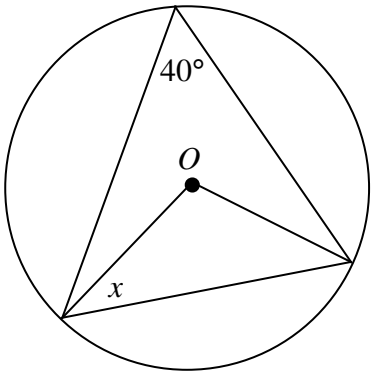
(A) $x \geq 3$

(B) $x \leq 0, x \geq 3$

(C) $0 \leq x \leq 3$

(D) $x \leq 4, x \geq 12$

2 In the following diagram, what is the value of x?



(A) 30°

(B) 40°

(C) 50°

(D) 60°

3 What does $P(x) = 4x^3 + 4x^2 - 11x - 6$ factorise to?

(A) $(4x - 1)(x + 6)(x + 1)$

(B) $(2x - 1)(2x + 3)(x + 2)$

(C) $(4x + 1)(x - 6)(x + 1)$

(D) $(2x + 1)(2x - 3)(x + 2)$

4 What is the derivative of $\log_e(\cos x)$?

(A) $-\sin x \cos x$

(B) $-\tan x$

(C) $-\cot x$

(D) $-\cos^2 x$

5 Which of the following equates to the expression $\dfrac{1 + e^{3x}}{1 + e^{x}}$?

(A) $1 + e^x + e^{2x}$

(B) $1 - e^x + e^{2x}$

(C) $1 + e^{2x}$

(D) $1 - e^x$

6 A committee of three is to be selected from a group comprising of four boys and seven girls. How many possible combinations are there to form this committee if at least two of the members are girls?

(A) ${}^{7}C_{3} + {}^{7}C_{2} \times {}^{4}C_{1}$

(B) ${}^{4}C_{3} + {}^{4}C_{2} \times {}^{7}C_{1}$

(C) ${}^{3}C_{1} \times {}^{2}C_{1} \times {}^{1}C_{1}$

(D) ${}^{7}C_{3}$

7 What is the domain and range of the inverse function $y = 2\tan^{-1} 3x$?

(A) Domain: $-\pi \leq x \leq \pi$; Range: All real y.

(B) Domain: $-3\pi \leq x \leq 3\pi$; Range: All real y.

(C) Domain: All real x; Range: $-\pi \leq y \leq \pi$.

(D) Domain: All real x; Range: $-3\pi \leq y \leq 3\pi$.

8 A spherical balloon was slowly inflated. At the point where its radius is 2 cm, the rate of change of its radius is 3 cm/s. What is the rate of change of its volume $\frac{dV}{dt}$ at this point?

Note: Volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

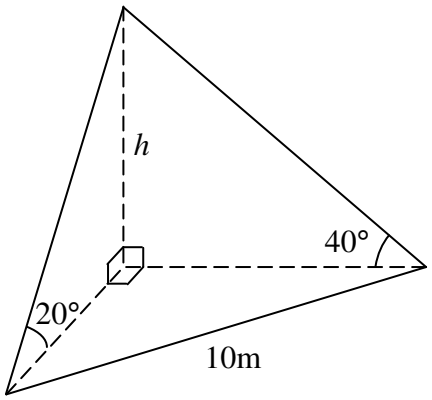
(A) $\frac{dV}{dt} = 4\pi \text{ cm}^3/\text{s}$

(B) $\frac{dV}{dt} = 12\pi \text{ cm}^3/\text{s}$

(C) $\frac{dV}{dt} = 16\pi \text{ cm}^3/\text{s}$

(D) $\frac{dV}{dt} = 48\pi \text{ cm}^3/\text{s}$

9 What is the value of h in the following diagram?



(A) $h = \dfrac{10}{\sqrt{\cot^2 20^\circ + \cot^2 40^\circ}}$

(B) $h = 10\sqrt{\cot^2 20^\circ + \cot^2 40^\circ}$

(C) $h = \dfrac{10}{\sqrt{\tan^2 20^\circ + \tan^2 40^\circ}}$

(D) $h = 10\sqrt{\tan^2 20^\circ + \tan^2 40^\circ}$

10 Differentiate with respect to x: $2\sin^{-1}\left(\sqrt{x}\right) - \sin^{-1}(2x - 1)$.

(A) 2

(B) $\dfrac{1}{\sqrt{1 - x^2}}$

(C) $\dfrac{2}{\sqrt{1 - x^2}}$

(D) 0

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question on a NEW page on your OWN PAPER.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW page on your OWN PAPER.

(a) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 5x}{4}$. **2**

(b) Differentiate $\tan^{-1}\left(x^2\right)$ with respect to x. **2**

(c) Find $\int_0^{\frac{\pi}{24}} \sin^2 3x \, dx$. **2**

(d) Solve for x: $\dfrac{4x+5}{x-2} \geq 1$. **3**

(e) Sketch the curve $y = 3\cos^{-1}\dfrac{x}{2} + \pi$. **3**

(f) Find the exact value of $\cos\left(2\tan^{-1}\dfrac{1}{3}\right)$. **3**

Question 12 (15 marks) Use a NEW page on your OWN PAPER.

(a) Find the term independent of x in the expansion of $\left(\frac{x^2}{4} - \frac{2}{x^3}\right)^{10}$. **2**

(b) Use the substitution $u = \sqrt{x}$ to evaluate $\int \frac{1}{1 + \sqrt{x}}\, dx$. **3**

(c) A group of 6 girls and 3 boys sat around a circular table. How many possible seating arrangements are there if:

(i) No restrictions applied. **1**

(ii) Three particular girls wanted to sit together next to each other. **2**

(d) The function $f(x) = \log_e x - \tan x$ has a root close to $x = 4$. Taking $x = 4$ as a first approximation, use one application of Newton's method to find a second approximation to the zero. Give your answer correct to two decimal places. **2**

(e) A particle moves along a straight line. Its displacement of x metres after t seconds is given by the formula: **2**

$$x = 3\sin 2t + 4\cos 2t.$$

Show that the particle moves with simple harmonic motion.

(f) Use mathematical induction to prove for all integers $n \geq 2$: **3**

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\ldots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}.$$

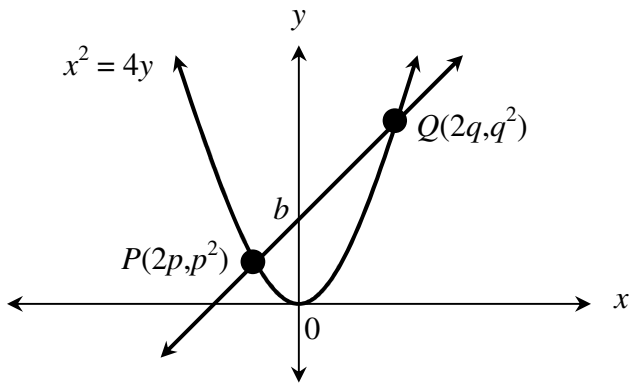
Question 13 (15 marks) Use a NEW page on your OWN PAPER.

(a) The polynomial $P(x) = x^3 - 4x^2 + 2x - 5$ has roots α, β and γ.

(i) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. **1**

(ii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. **1**

(b) The straight line $y = x + b$ (where $b > 0$) meets the parabola $x^2 = 4y$ at $P(2p,p^2)$ and $Q(2q,q^2)$.



(i) Show that $p + q = 2$. **1**

(ii) The equation of the normal at P is $x + py = 2p + p^3$ [Do NOT prove this]. **2**
Show that normals at P and Q intersect at the point:

$$N[-pq(p + q), p^2 + q^2 + pq + 2].$$

(iii) Hence, or otherwise, show that the locus of N follows the linear equation: **2**

$$x - 2y + 12 = 0.$$

(c) In terms of its displacement (x), a particle's acceleration is given by the formula:

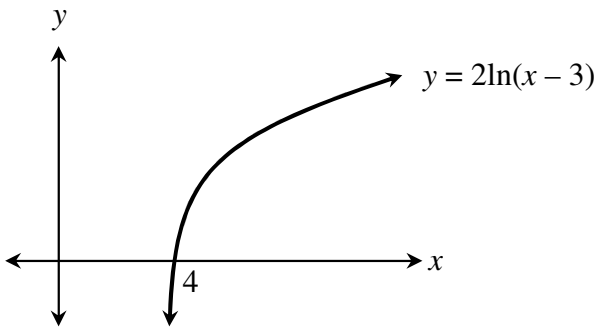
$$\ddot{x} = 4x - 4.$$

Initially, the particle is 2 metres to the right of the origin with a velocity of 2 ms^{-1}.

(i) Show that $v = 2(x - 1)$. **2**

(ii) Show that $x = e^{2t} + 1$. **2**

(d) The following is a sketch of the function $y = 2\ln(x - 3)$:



Using the sketch, or otherwise,:

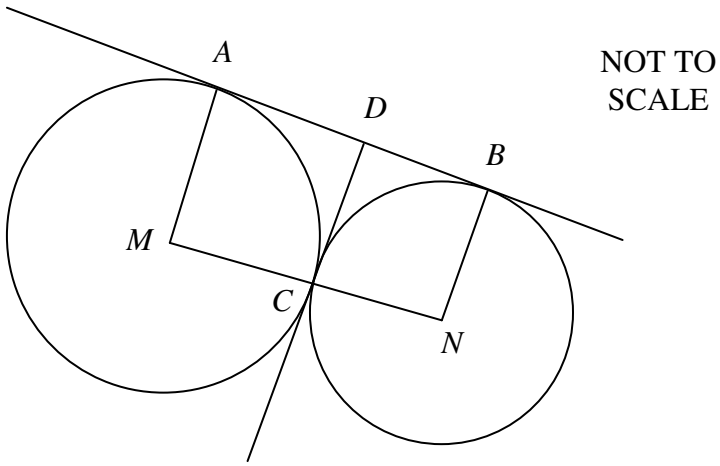
(i) Find the area bound by the curve $y = 2\ln(x - 3)$, the x-axis and the line $x = 7$. **2**

(ii) Find the volume of the solid formed when the curve $y = 2\ln(x - 3)$ is rotated about the y-axis between $y = 0$ and $y = 2\ln 4$. **2**

Question 14 (15 marks) Use a NEW page on your OWN PAPER.

(a) Solve for θ, where $0 \le \theta \le 2\pi$: $\sin\theta + 2\sin\theta\cos\theta + \cos\theta = 1$. **3**

(b) In the diagram below, *MCN* is a straight line. Circles are drawn with centre *M* and radius *MC*, and centre *N* and radius *NC*. *AB* is a common tangent to the circles with points of contact at *A* and *B* respectively. *CD* is a common tangent at *C* and meets *AB* at *D*.



(i) Explain why *AMCD* and *BNCD* are cyclic quadrilaterals. **1**

(ii) Show that ΔACD is similar to ΔCBN. **3**

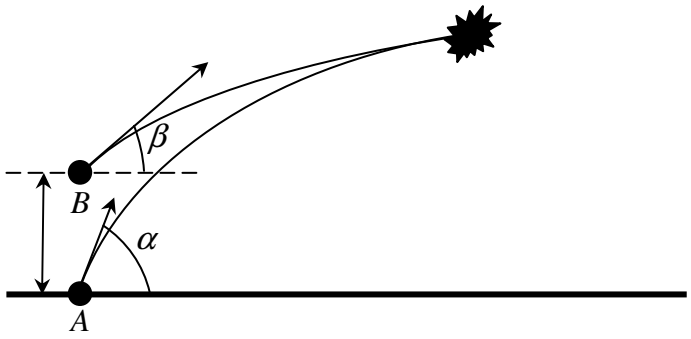
(iii) Show that *MD* is parallel to *CB*. **2**

(c) Given the identity $(1+x)^n = {}^nC_0 + {}^nC_1 x^1 + {}^nC_2 x^2 + {}^nC_3 x^3 + ... + {}^nC_n x^n$ **3**
show, using the binomial theorem, that

$$1^2\binom{n}{1} + 2^2\binom{n}{2} + 3^2\binom{n}{3} + ... + n^2\binom{n}{n} = n(n+1)2^{n-2}$$

where n is a positive integer.

(d) Two rockets, A and B, were launched at the same time with velocities of U ms^{-1} and V ms^{-1},and angles of elevation of α and β respectively, where $\beta < \alpha$. A is launched from the ground while B is launched h metres vertically above A. Both rockets collide after T seconds after the launch. **3**



After t seconds, the horizontal (x) and vertical (y) displacements of each rocket is given as follows (**DO NOT PROVE THESE**):

Rocket A: $x = U\cos\alpha t$ and $y = -\frac{gt^2}{2} + U\sin\alpha t$

Rocket B: $x = V\cos\beta t$ and $y = -\frac{gt^2}{2} + V\sin\beta t + h$

where gravity is g ms^{-2}.

Show that $T = \frac{h\cos\beta}{U\sin(\alpha - \beta)}$.

End of paper.