NExT EDUCATION

2017

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used

Total marks – 100

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 7 – 15

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

---

1 What does i^{2017} equal to?

(A) 1

(B) -1

(C) i

(D) $-i$

2 What is the value of $\int_0^{\frac{2}{3}} \frac{1}{9x^2+4}\,dx$?

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{24}$

(D) $\frac{\pi}{36}$

3 What are the complex solutions for z in the equation $z^2 - 4z + 6 = 0$?

(A) $z = 1 + i$

(B) $z = 2 + 2i$

(C) $z = 2 \pm i\sqrt{2}$

(D) $z = 2 \pm i\sqrt{5}$

4 Find the value of the eccentricity (e) of the following equation: $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$.

(A) $e = \dfrac{4}{3}$

(B) $e = \dfrac{5}{3}$

(C) $e = \dfrac{3}{5}$

(D) $e = \dfrac{3}{4}$

5 Which of the following are the square roots of the complex number $5 - 12i$?

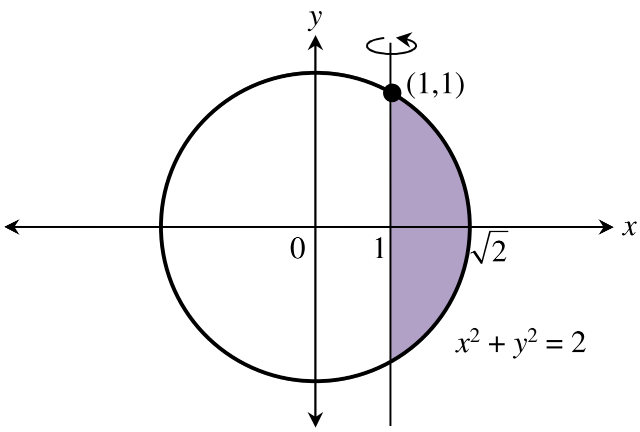
(A) $2 - 3i, -2 + 3i$

(B) $2 + i, -2 - i$

(C) $-3 + 2i, 3 - 2i$

(D) $13 + 13i, -13 - 13i$

6 The diagram shows a circle with equation of $x^2 + y^2 = 2$, where the shaded area is a minor segment bound by the circle and the line $x = 1$.



If the shaded area is rotated about the line $x = 1$, using the method of cylindrical shells, what is an expression for the volume produced?

(A) $2\pi\int_0^1 \left(\sqrt{2-y^2}-1\right)^2 \, dy$

(B) $2\pi\int_0^1 y^2 - 1 \, dy$

(C) $4\pi\int_1^{\sqrt{2}} (x-1)\sqrt{2-x^2} \, dx$

(D) $4\pi\int_1^{\sqrt{2}} x\sqrt{2-x^2} \, dx$

7 The equation $x^3 - x^2 - 3x + 2 = 0$ has roots $x = \alpha, \beta$ and γ.
Find the value of $(\alpha+\beta)(\alpha+\gamma)(\beta+\gamma)$.

(A) -1

(B) 1

(C) 5

(D) 7

8 On an Argand diagram, the points A and B are represented by the complex numbers z_1 and z_2 respectively. Which of the following best describes the locus of

$$\text{Arg}(z - z_1) - \text{Arg}(z - z_2) = \theta\,?$$

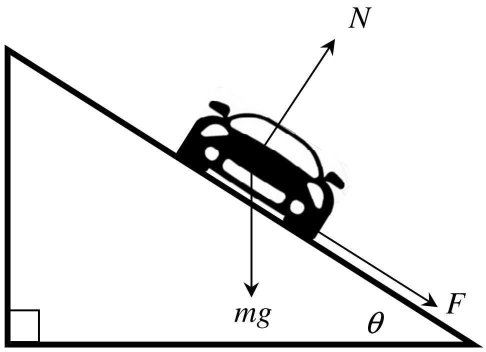
(A) A ray at the point represented by $(z_1 - z_2)$ with angle of θ.

(B) A circle with centre at the point represented by $(z_1 - z_2)$ and radius of $|z_1 - z_2|$.

(C) A circle travelling anti-clockwise from A to B, terminating at A and B (excluding those points).

(D) A circle travelling anti-clockwise from B to A, terminating at A and B (excluding those points).

9 A vehicle of mass m kg moving with velocity v m/s is rounding a curve of radius r metres banked at an angle of θ. A lateral (sideways) friction force F is acting between its tyres and the road, and a normal force N is acting on the tyres. Gravity is g m/s^2,



At what velocity would the vehicle experience no friction force (i.e. would not slip)?

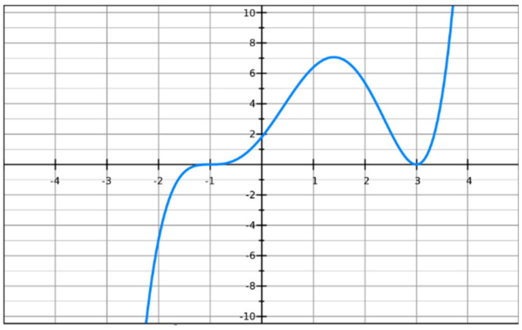
(A) $v = mg\cos\theta$

(B) $v = g\sec\theta$

(C) $v = \sqrt{\dfrac{rg}{\sin\theta}}$

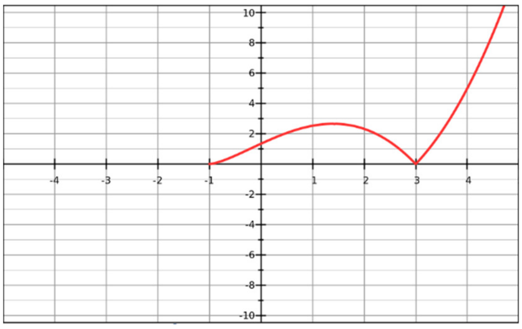
(D) $v = \sqrt{rg\tan\theta}$

10 The following diagram shows the graph of $y=\dfrac{(x+1)^3(x-3)^2}{5}$:

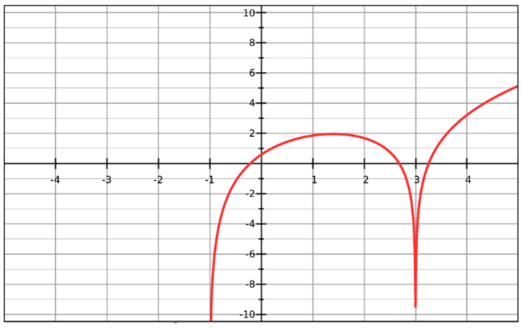


Which of the following graphs best represents $y=\ln\left[\dfrac{(x+1)^3(x-3)^2}{5}\right]$?

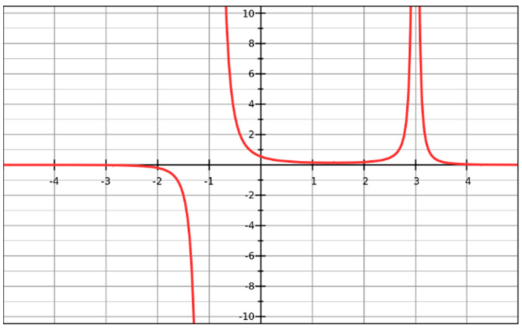
(A)



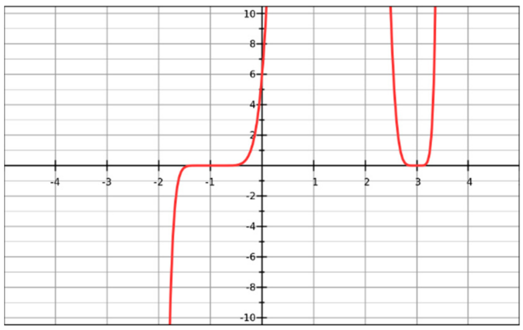
(B)



(C)



(D)



Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question on a NEW page on your OWN PAPER.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW page on your OWN PAPER.

(a) Find $\int \left(5+4x-x^2\right)^{-\frac{1}{2}} dx$. **2**

(b) Let $w = 3 - \sqrt{3}i$.

(i) Express w in modulus-argument form. **1**

(ii) Express w^{12} in modulus-argument form. **2**

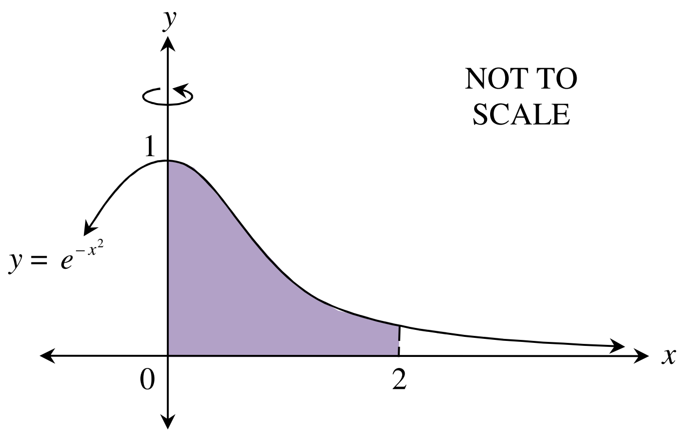
(c) Using the substitution $t = \tan\frac{x}{2}$, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin x}\, dx$. **3**

(d) (i) Find real numbers a, b and c such that **2**

$$\frac{4x^2-17x+25}{(x+2)(x-3)^2} = \frac{a}{x+2} + \frac{b}{x-3} + \frac{c}{(x-3)^2}.$$

(ii) Hence, or otherwise, find $\int \frac{4x^2-17x+25}{(x+2)(x-3)^2}\, dx$. **2**

(e) In the diagram below, the shaded area shows the area enclosed between the curve $y = e^{-x^2}$ and the x-axis, between $x = 0$ and $x = 2$. **3**



Using the method of cylindrical shells, find the volume of the solid formed when the shaded region in the diagram is rotated about the y-axis.

Question 12 (15 marks) Use a NEW page on your OWN PAPER.

(a) Sketch the following on different complex planes labelling all key features:

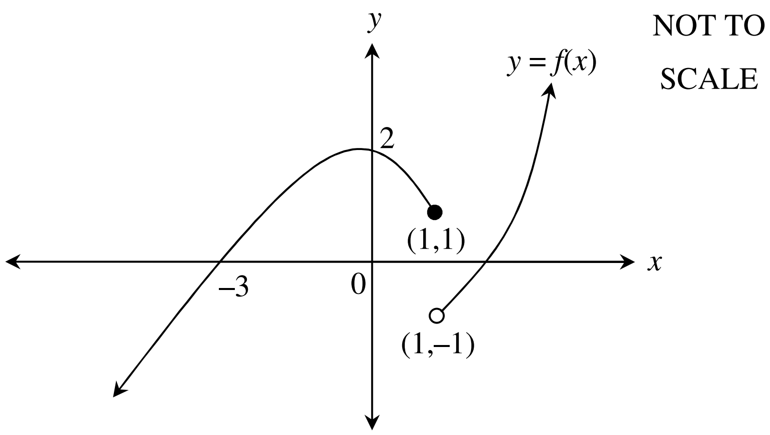
(i) $|z-2| \le 1$. **1**

(ii) $\dfrac{\pi}{4} \le \operatorname{Arg}(z + 1 - i) \le \dfrac{\pi}{2}$. **2**

(iii) $\operatorname{Re}(z) = |z|$. **2**

(b) Evaluate $\displaystyle\int_0^{\frac{a}{2}} x^2\sqrt{a^2 - x^2}\ dx$. **3**

(c) The diagram shows the graph of a function $f(x)$.



Sketch the following curves on separate half-page diagrams.

(i) $y = |f(x)|$ **1**

(ii) $y = [f(x)]^2$ **2**

(iii) $y \times f(x) = 1$ **2**

(iv) $y = e^{f(x)}$ **2**

Question 13 (15 marks) Use a NEW page on your OWN PAPER.

(a) Use integration by parts to find $\int x^2 e^{-x}\,dx$. **3**

(b) If α, β and γ are the roots of the equation $x^3 - x^2 - 2x + 4 = 0$,

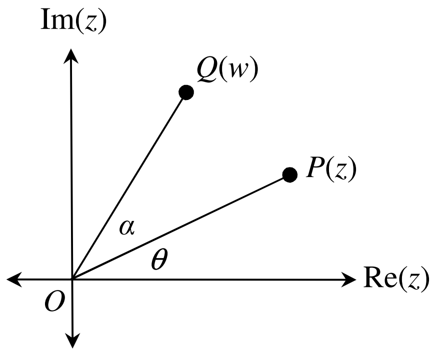
(i) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. **1**

(ii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. **2**

(iii) Find an equation with roots of $1 - \alpha$, $1 - \beta$ and $1 - \gamma$. **1**

(iv) Find an equation with roots of $\dfrac{\alpha+\beta}{\gamma}$, $\dfrac{\beta+\gamma}{\alpha}$ and $\dfrac{\alpha+\gamma}{\beta}$. **3**

(c) On the Argand diagram, the points P and Q are represented by the complex numbers z and w respectively, as shown in the diagram below.



Let the complex number $z = \cos\theta + i\sin\theta$, $0 < \theta < \dfrac{\pi}{4}$. In the diagram, $OP = OQ$ and $\angle QOP = \alpha$, where $0 < \alpha < \dfrac{\pi}{4}$.

(i) Express the complex number w in modulus-argument form. **1**

(ii) Show that $z\overline{w} = \cos\alpha - i\sin\alpha$. **2**

(iii) By considering ΔOPQ, or otherwise, deduce that $\cos\left(\dfrac{\alpha}{2}\right) = -\dfrac{\mathrm{Im}(z\overline{w})}{|z-w|}$. **2**

Question 14 (15 marks) Use a NEW page on your OWN PAPER.

(a) Find the equation of the tangent to the curve $x^2 - xy + y^3 = 5$ at the point (2,–1). **3**

(b) (i) Let $I_n = \int_1^2 x(\ln x)^n \, dx$ for integers $n \geq 0$. **3**

Show that $I_n = 2(\ln 2)^n - \dfrac{n}{2} I_{n-1}$, $n \geq 1$.

(ii) Hence, or otherwise, evaluate: $\int_1^2 x(\ln x)^3 \, dx$. **2**

(c) (i) Using De Moivre's theorem, show that: **2**

$$\tan 3\theta = \frac{\tan^3\theta - 3\tan\theta}{3\tan^2\theta - 1}.$$

(ii) Hence or otherwise, find all the roots of $x^3 - 3x^2 - 3x + 1 = 0$. **3**

(iii) Show that $\tan\dfrac{\pi}{12} + \tan\dfrac{5\pi}{12} = 4$. **2**

Question 15 (15 marks) Use a NEW page on your OWN PAPER.

(a) An object, P, of mass m kg is released from a point A and falls vertically towards a point on the ground B. At the point of release another object, Q, with identical mass is projected vertically upwards from B with initial velocity that is twice the terminal velocity of object P.

Both objects are subject to air resistance of mkv, where v m/s is the velocity of the objects and k is a constant. Assume gravity is g m/s^2.

(i) Show that the terminal velocity of object P is $\dfrac{g}{k}$. **1**

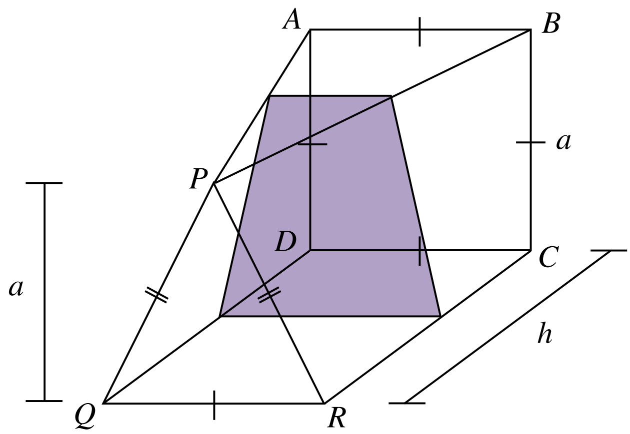
(ii) For object Q, show that the time of flight, t in seconds, is given by the equation: **3**

$$t = \frac{1}{k}\log_e\left(\frac{3g}{g+kv}\right).$$

(iii) The objects P and Q collide in mid-air when object P reaches 30% of its terminal velocity. Find the velocity of Q when the collide in terms of g and k. **2**

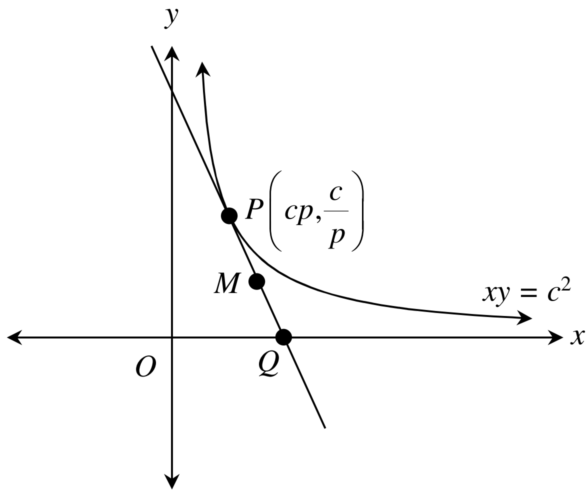
(b) A solid $ABCDPQR$ is formed such that $ABCD$ is a square with sides of a metres and PQR is an Isosceles triangle with $PQ = PR$. The base, QR, and perpendicular height, PM, of triangle PQR are both a metres in length. **4**

The cross-sections perpendicular to the base $QRCD$ are trapeziums. A typical cross-section is shown shaded in the diagram.



If the solid is h metres deep, find the volume of the solid in terms of a and h.

(c) The point $P\left(cp, \frac{c}{p}\right)$ lies on the rectangular hyperbola $xy = c^2$, as shown in the diagram below. The tangent at P cuts the x-axis at Q. The point M is the midpoint of PQ.



(i) Show that the equation of the tangent at P is $x + p^2y = 2cp$. **1**

(ii) Find the coordinates of Q. **1**

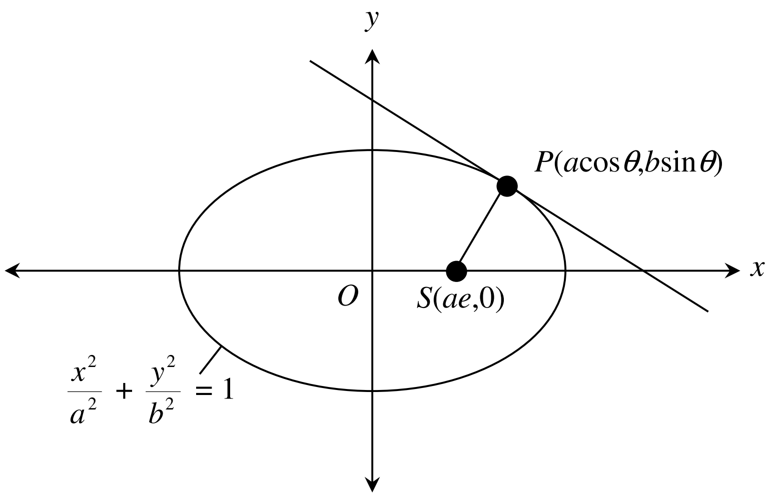
(iii) Find the coordinates of M, and show that its locus is a hyperbola. **3**

Question 16 (15 marks) Use a NEW page on your OWN PAPER.

(a) (i) Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. **1**

(ii) Hence, or otherwise, evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$. **3**

(b) $P(a\cos\theta, b\sin\theta)$ is a point that lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, as shown in the diagram below.



The focus of the ellipse is the point $S(ae,0)$, where e is the eccentricity of the ellipse.

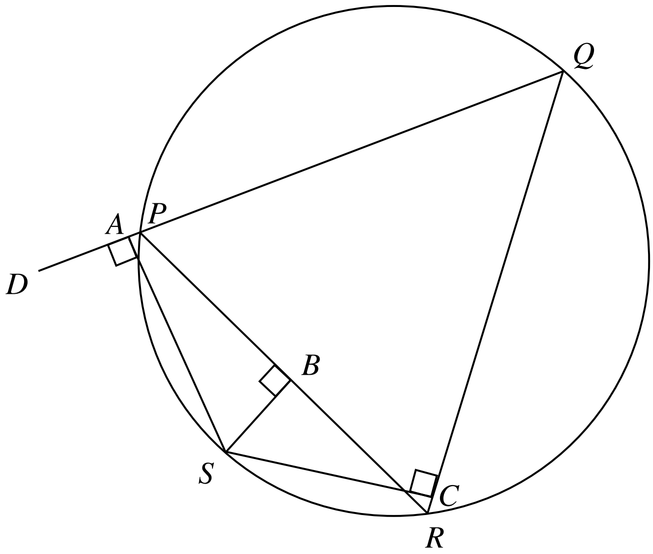
(i) Find the gradient of the tangent at P. **1**

(ii) Show that the product of the gradient of the interval SP and the gradient of the tangent at P is: **2**

$$\frac{\cos\theta\left(1-e^2\right)}{e-\cos\theta}$$

(iii) Prove that the interval SP is never perpendicular to the tangent at P, provided that $\theta \neq 0$ or π. **2**

(c) In the diagram, $PQRS$ is a cyclic quadrilateral and the points A, B and C are perpendiculars drawn S to QP produced, PR and QR respectively.



Copy this diagram.

(i) Prove that $\angle SBA = \angle SPA$. **2**

(ii) Prove that $\angle SBC + \angle SRC = 180°$. **2**

(iii) Hence, or otherwise, prove that points A, B and C are collinear. **2**

End of paper.