Student details

Name: ______________________

Mark: ______________________

2021

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 6 – 12

60 marks

- Attempt Questions 11 – 28
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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1 What are the solutions for x in the equation $|x+1| = 3x + 7$.

(A) $x = 3, 2$

(B) $x = -3, -2$

(C) $x = 2$ only

(D) $x = -2$ only

2 The polynomial $P(x) = x^3 + 6x^2 - x - 2$ has roots α, β and γ.

What is the value of $\alpha^2 + \beta^2 + \gamma^2$?

(A) 6

(B) 34

(C) 36

(D) 38

3 What is the derivative of $\sin(\sin x)$?

(A) $\cos(\cos x)$

(B) $(\cos x)^2$

(C) $\sin(\cos x)$

(D) $\cos(\sin x)\cos x$

4 If $(x - 2)$ is a factor of the polynomial $P(x) = 3x^3 + mx^2 + nx + 12$, what can be the values for m and n?

(A) $m = 4, n = -3$

(B) $m = 15, n = -12$

(C) $m = 9, n = -28$

(D) $m = 11, n = -40$

5 Which of the following scenarios is an appropriate application for the differential equation $\frac{dM}{dt} = -k(M - B)$, where k and B are positive constants?

(A) The unrestricted population growth of a colony of insects.

(B) The defrosting of a frozen apple pie in an oven.

(C) The viral infection of a restricted population on a secluded island.

(D) The cooling of a warm cup of tea left in a room with constant temperature.

6 A committee of three is to be selected from a group comprising of four boys and seven girls. How many possible combinations are there to form this committee if at least two of the members are girls?

(A) $^7C_3 + {}^7C_2 \times {}^4C_1$

(B) $^4C_3 + {}^4C_2 \times {}^7C_1$

(C) $^3C_1 \times {}^2C_1 \times {}^1C_1$

(D) $^7C_3$

7 What is the domain and range of the function $y = 6\cos^{-1}(2x-1) - \pi$?

(A) Domain: $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$; Range: $y \in [-\pi, 6\pi]$

(B) Domain: $x \in [0,1]$; Range: $y \in [-\pi, 5\pi]$

(C) Domain: $x \in [1,6]$; Range: $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(D) Domain: $x \in \left[-\frac{1}{2}, \frac{3}{2}\right]$; Range: $y \in [0, 2\pi]$

8 Which of the following is equivalent to $\sin\left(2\tan^{-1}\frac{a}{b}\right)$?

(A) $\dfrac{ab}{\sqrt{a^2 + b^2}}$

(B) $\dfrac{2ab}{a^2 + b^2}$

(C) $\dfrac{1}{\sqrt{b^2 - a^2}}$

(D) $\dfrac{1}{a^2 - b^2}$

9 Four letters in the word "TREEHOUSE" is selected at random to form a new 'word'. How many arrangements are possible?

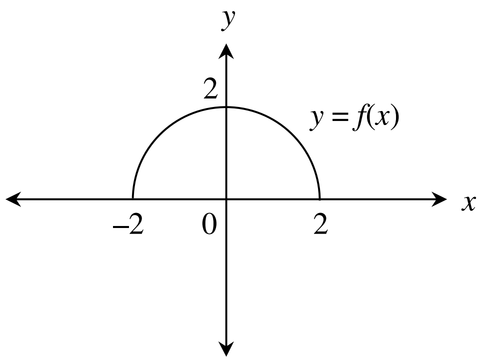
(A) 24

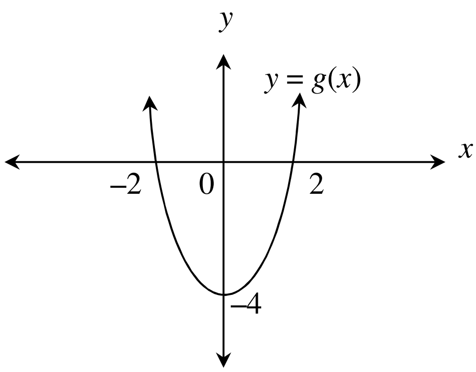
(B) 754

(C) 1044

(D) 60480

10 The graphs of $y = f(x)$ and $y = g(x)$ are given below:





What is the domain of $y = f\left[g(x)\right]$?

(A) Domain: $x \in \left[-2, 2\right]$

(B) Domain: $x \in \left[-\sqrt{3}, -\sqrt{2}\right] \cup \left[\sqrt{2}, \sqrt{3}\right]$

(C) Domain: $x \in \left[-\sqrt{6}, -\sqrt{2}\right] \cup \left[\sqrt{2}, \sqrt{6}\right]$

(D) Domain: $x \in \left[-\sqrt{3}, -1\right] \cup \left[1, \sqrt{3}\right]$

Section II

60 marks
Attempt Questions 11–28

In Questions 11–28, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Solve for x, expressing you solution in set notation: $\dfrac{2-3x}{x+5} \leq 1$. **2**

Question 12

Differentiate $\sin^{-1}(\log_e x)$ **2**

Question 13

Find the term independent of x in the expansion of $\left(5x - \dfrac{2}{x}\right)^8$. **2**

Question 14

Find the exact value of $\cos\left(2\cot^{-1}\dfrac{4}{3}\right)$. **2**

Question 15

Use the substitution $u = e^x$ to evaluate $\displaystyle\int \frac{1}{e^x + e^{-x}}\,dx$. **2**

Question 16

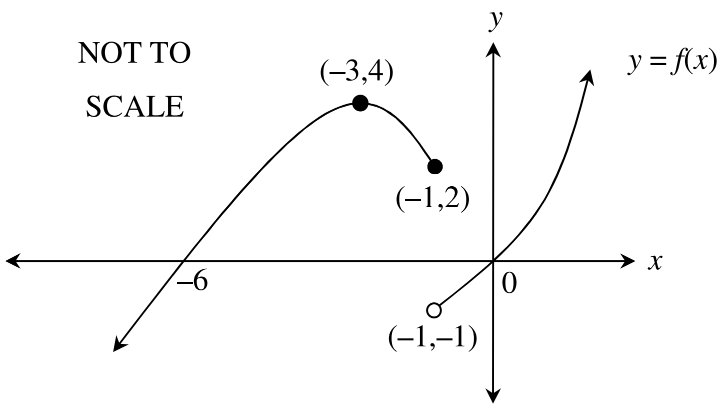
On a Cartesian plane, the position vectors at points A and B are $\underset{\sim}{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\underset{\sim}{b} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$ respectively.

(a) Find $|\underset{\sim}{a}|$. **1**

(b) Find the vector projection of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$. **2**

Question 17

The diagram shows the graph of a function $f(x)$.



Sketch the following curves on <u>separate half-page diagrams</u>.

(a) $y = f(-x)$ **1**

(b) $y = |f(x)|$ **1**

(c) $|y| = f(|x|)$ **2**

(d) $y = \dfrac{1}{f(x)}$ **3**

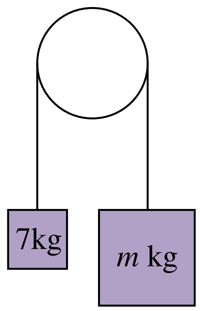
Question 18

(a) Sketch the graph $y = 4\sin 2x$ for $x \in [0, 2\pi]$, labelling all key features. **2**

(b) Hence, or otherwise, find the volume of the solid formed when the region bound by the curve $y = 4\sin 2x$, the x-axis and the line $x = \frac{\pi}{4}$ is rotated about the x-axis. **2**

Question 19

Two objects with mass m kilograms and 7 kilograms were connected via a light inextensible string in a pulley system, as shown in the diagram below:



The system accelerated such that the 7 kilogram mass moved upwards at a rate of 2.5 m/s^2. Assuming gravity of 9.8 m/s^2,

(a) Find the amount of tension in the string. **2**

(b) Find the value of m, rounding your solution to one decimal place. **2**

Question 20

Find the solution to the differential equation: $\operatorname{cosec} x \dfrac{dy}{dx} = \dfrac{e^{\cos x}}{6y}$, given $y = 0$ when $x = 0$. **2**

Question 21

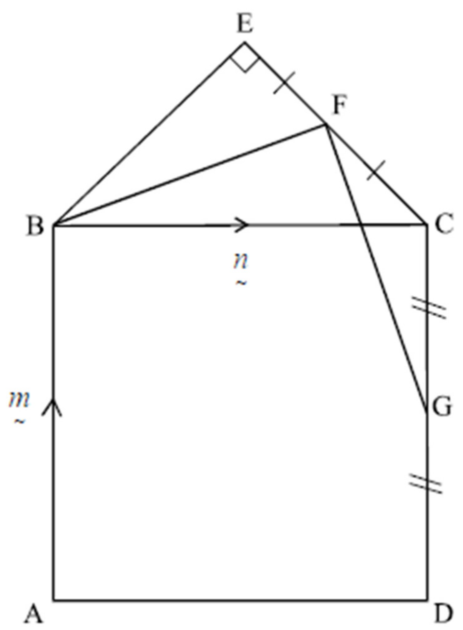
The polynomial $P(x) = 5x^4 + 41x^3 + 99x^2 + 27x - 108$ has a root of multiplicity 3.

(a) Find the root of multiplicity 3. **2**

(b) Hence, or otherwise, fully factorise $P(x) = 5x^4 + 41x^3 + 99x^2 + 27x - 108$. **1**

Question 22

$ABCD$ is a square where $\overrightarrow{AB} = \underset{\sim}{m}$ and $\overrightarrow{BC} = \underset{\sim}{n}$. BCE is a right-angled isosceles triangle. The point F is the midpoint of CE and the point G is the midpoint of CD, as shown in the diagram below. **3**



Using vectors, prove that $\angle BFG$ is a right angle.

Question 23

(a) Show that the term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is: **2**

$(-1)^n \times \binom{2n}{n}$

(b) Show that: $(1+x)^{2n}\left(1 - \frac{1}{x}\right)^{2n} = \left(x - \frac{1}{x}\right)^{2n}$ **1**

(c) Hence, or otherwise, show that: **2**

$$\binom{2n}{0}^2 - \binom{2n}{1}^2 + \binom{2n}{2}^2 - \ldots + \binom{2n}{2n}^2 = (-1)^n \times \binom{2n}{n}$$

Question 24

A tank contained 8L of water in which 10g of salt was dissolved into it and mixed through. The tank had a pipe pumping in a solution at a rate of 2 litres per minute, where the solution coming in had a salt concentration of 5 grams per litre. The resultant solution was mixed thoroughly before being released via an out-valve at a rate of 2 litres per minute.

(a) Show that the amount of salt M in the tank after t minutes follows the differential equation: **1**

$$\frac{dM}{dt} = \frac{40 - M}{4}.$$

(b) Find the solution to the differential equation $\frac{dM}{dt} = \frac{40 - M}{4}$ in the form $M = A - Be^{-kt}$, where A, B and k are constants. **2**

(c) Find the salt concentration in the tank after 6 minutes, rounding your solution to three decimal places. **1**

Question 25

(a) Show that: $\cot\theta + \frac{1}{2}\tan\frac{\theta}{2} = \frac{1}{2}\cot\frac{\theta}{2}$. **2**

(b) Hence, or otherwise, prove by mathematical induction for $n \in \mathbb{Z}^+$: **3**

$$\sum_{r=1}^{n} \frac{1}{2^{r-1}} \tan\frac{x}{2^r} = \frac{1}{2^{n-1}}\cot\frac{x}{2^n} - 2\cot x.$$

Question 26

In a factory producing vases, faults were found at an alarming rate of 3 in every 10, where the number of faults followed a *binomial distribution*. A random batch of 20 vases was selected as part of a quality assurance process.

(a) Find the probability that 11 of the 20 vases were faulty, rounding your solution to three significant figures. **2**

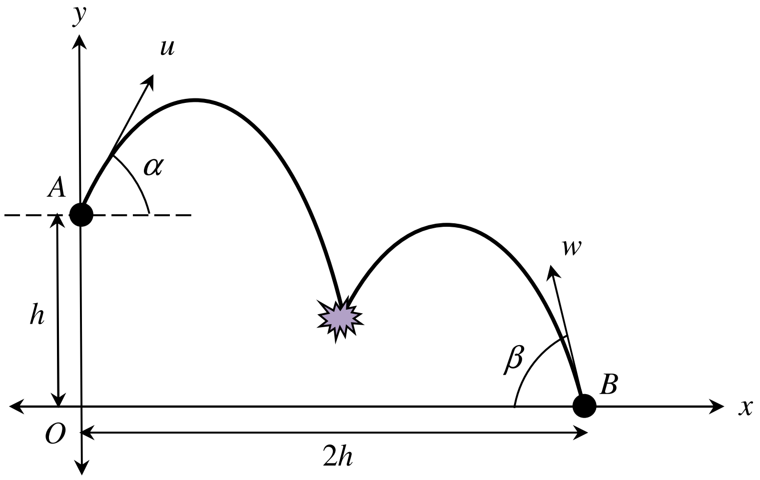
(b) Using the standard normal table of values below, approximate the probability of 11 faulty vases using the normal distribution, rounding your solution to three significant figures. **2**

z	First Decimal Place									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.0	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.0	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.0	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

(c) Hence, using the rounded solutions in part (a) and (b), determine the percentage error in using the normal approximation, rounding your solution to three significant figures. **1**

Question 27

An object was projected into the air from a point A that was h metres above O with initial velocity u m/s at an angle of α from the horizontal. At the same time, a particle was projected into the air from point B that was $2h$ metres away from O along the horizontal plane with initial velocity w m/s at an angle of β from the horizontal. The two particles collide in mid-air T seconds after projection, as shown in the diagram below.



Assuming gravity is g m/s^2, after t seconds the equation of motion for the object projected at point A is given by the following (**DO NOT PROVE THESE**):

$$x_A = u\cos\alpha t \qquad \text{and} \qquad y_A = -\frac{gt^2}{2} + u\sin\alpha t + h$$

(a) For the particle projected at point B, write down the expressions for the horizontal distance x_B from O and its vertical distance y_B after t seconds. **2**

(b) Hence, or otherwise, show that: $\frac{u}{w} = \frac{2\sin\beta - \cos\beta}{\cos\alpha + 2\sin\alpha}$. **2**

Question 28

Find the Cartesian equation of the following parametric equations: **2**

$$\begin{cases} x = 3\cos\theta + \sin\theta \\ y = 2\cos\theta - \sin\theta \end{cases}$$

End of paper.