NExT EDUCATION

2018

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used

Total marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 13

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

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1 What is the value of $|-2| - |-5|$?

(A) 7

(B) –3

(C) –7

(D) 3

2 What is the value of x: $8^x = 32$

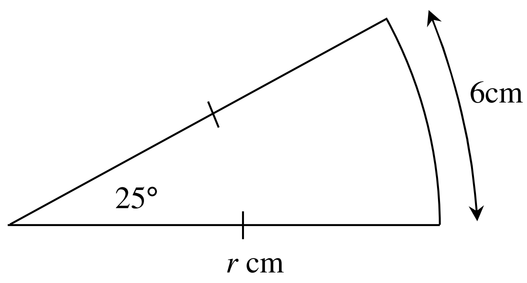
(A) $x = 3$

(B) $x = \frac{2}{5}$

(C) $x = \frac{4}{3}$

(D) $x = \frac{5}{3}$

3 Which of the following is equal to the value of r?



(A) $r = 6 \times \frac{180}{25\pi}$

(B) $r = 6 \times 25\pi$

(C) $r = 6 \times \frac{25}{180}\pi$

(D) $r = \frac{180\pi}{6 \times 25}$

4 The equation $x^2 + 4x - 1 = 0$ has roots $x = \alpha$ and $x = \beta$. What is the value of $\alpha^2 + \beta^2$?

(A) 5

(B) 14

(C) 18

(D) –7

5 Which of the following is a solution for x in the equation: $\sqrt{2}\cos x + 1 = 0$

(A) $x = \frac{\pi}{4}$

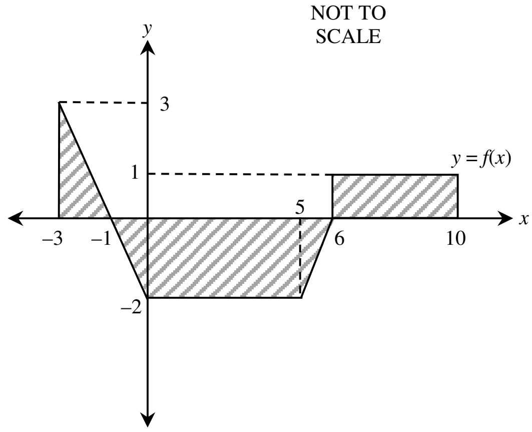
(B) $x = \frac{\pi}{6}$

(C) $x = \frac{5\pi}{4}$

(D) $x = \frac{2\pi}{3}$

6 The following diagram shows the graph $y = f(x)$.

Using the graph, evaluate: $\int_{-3}^{10} f(x)\,dx$.



(A) 9

(B) 3

(C) –2

(D) –5

7 A geometric series with a first term of 8 has a limiting sum of 12. What is the value of its common ratio?

(A) $\frac{1}{6}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

8 What is the domain of the function $f(x) = \dfrac{1}{\sqrt{x-6}}$?

(A) All real x

(B) $0 < x < 6$

(C) $x \geq 6$

(D) $x > 6$

9 What is the amplitude and period of the equation $y = \frac{1}{3}\sin 4x$?

(A) Amplitude is $\frac{1}{3}$ and period is $\frac{\pi}{2}$

(B) Amplitude is $\frac{1}{3}$ and period is 2π

(C) Amplitude is 4 and period is $\frac{\pi}{3}$

(D) Amplitude is 4 and period is $\frac{3\pi}{2}$

10 Consider the point P where $x = a$ on the curve $y = f(x)$.

If $f'(a) = 0$ and $f''(a) < 0$, which of the following statements best describe the point P on the curve $y = f(x)$?

(A) P is a maximum stationary point.

(B) P is a minimum stationary point.

(C) P is an inflexion point.

(D) P is a horizontal inflexion point.

End of Section I.

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question on a NEW page on your OWN PAPER.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW page on your OWN PAPER.

(a) Simplify fully: $2\sqrt{27} - \sqrt{12} + 5\sqrt{3}$. **2**

(b) Evaluate $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$. **2**

(c) Solve for x: $\dfrac{x-2}{x+5} \geq 0$. **2**

(d) Differentiate the following with respect to x:

(i) $y = (4x - 5)^2$. **1**

(ii) $y = \dfrac{x^2}{\cos x}$. **2**

(iii) $y = \log_e\left(3 - x^3\right)$. **2**

(e) Find the limiting sum of the series: $6 - 2 + \dfrac{2}{3} - \dfrac{2}{9} + \ldots$ **2**

(f) If $\cos\theta = \dfrac{2}{7}$ and $\sin\theta < 0$, find the exact value of $\tan\theta$. **2**

End of Question 11.

Question 12 (15 marks) Use a NEW page on your OWN PAPER.

(a) Find:

(i) $\int \cos(2018x)\, dx$. **1**

(ii) $\int \frac{5x}{x^2 + 1}\, dx$. **2**

(iii) $\int \frac{5}{x} + e^{-3x}\, dx$. **2**

(b) Prove the identity: $\frac{\cos x}{1 - \sin x} - \sec x = \tan x$. **2**

(c) Find the equation of the tangent to the curve $y = (3x - 1)^2$ at the point where $x = 1$. **2**

(d) By using Simpson's Rule with five function values, estimate the value of $\int_1^5 \log_e x\, dx$. **2**

Round your solution to two decimal place.

(e) The chance of rain on a given day over the year in the rural town of Cowhop is 20%. What is the probability of at least one day of rain over three consecutive days? **2**

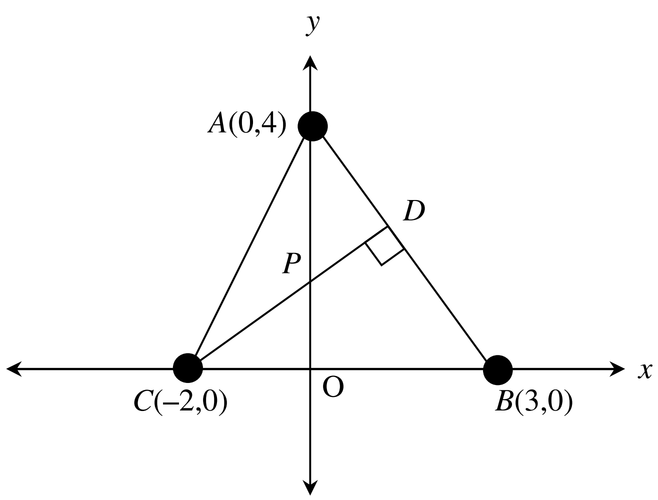
(f) Simplify: $\frac{2^{3x-1}}{8^{x-1}} \times \frac{25^y}{5^{2y+1}}$. **2**

End of Question 12.

Question 13 (15 marks) Use a NEW page on your OWN PAPER.

(a) Solve for x: $\log_2\left(\log_2 x\right) = 3$. **2**

(b) Triangle ABC lies on a Cartesian plane and has vertices $A(0,4)$, $B(3,0)$ and $C(-2,0)$, as shown in the diagram below.



AO and CD are the altitudes drawn from vertices A and C respectively.

(i) Find the gradient of the interval AB. **1**

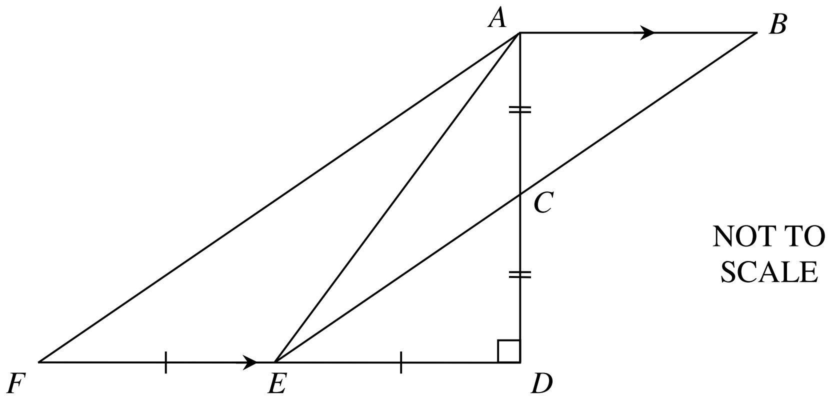
(ii) Show that the interval AB has equation $4x + 3y - 12 = 0$. **1**

(iii) Find the perpendicular distance between the AB and the point $C(-2,0)$. **2**

(iv) Find the equation of the altitude CD. **2**

(v) Hence, or otherwise, find the coordinates of the point P, the point of intersection of the altitudes AO and CD. **1**

(c)



In the diagram, $AB \parallel FD$, ADF is a right-angled triangle, C is the midpoint of AD and E is the midpoint of FD.

(i) Explain why $\angle CED = \angle ABC$. **1**

(ii) Prove that $\Delta CDE \equiv \Delta CAB$. **2**

(iii) Show that $AF = 2BC$. **2**

(iv) Show that $\angle ACB = \angle DAF$. **1**

End of Question 13.

Question 14 (15 marks) Use a NEW page on your OWN PAPER.

(a) The equation $x^2 - 6x - 3 = 0$ has roots $x = \alpha$ and β. Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$. **2**

(b) Solve for x: $x^{-4} - 3x^{-2} - 4 = 0$. **3**

(c) A particle moves along a straight line where its displacement x metres after time t seconds is given by the formula:

$$x = 1 + 3\sin 2t.$$

(i) In terms of t, find an expression for the particle's velocity v and acceleration a. **2**

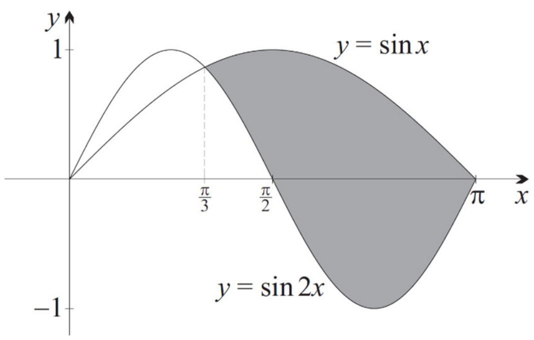
(ii) When does the particle first come to rest? **1**

(iii) What is the maximum displacement of the particle? **1**

(iv) What is the maximum velocity of the particle? **1**

(v) Sketch the graph of the particle's displacement against time for $0 \le t \le 2\pi$. **2**

(d) **3**



The diagram above shows the curves $y = \sin 2x$ and $y = \sin x$ for $0 \le x \le \pi$, intersecting at $x = 0$, $x = \dfrac{\pi}{3}$ and $x = \pi$. Find the exact area of the shaded region bounded by the two curves.

End of Question 14.

Question 15 (15 marks) Use a NEW page on your OWN PAPER.

(a) (i) Differentiate with respect to x: $x^2\log_e x$. **1**

(ii) Hence, or otherwise, find: $\int x \log_e x \, dx$. **2**

(b) Consider the function $y = 1 - 3x + x^3$ for the domain $-3 \leq x \leq 2$.

(i) Find the coordinates of the stationary points and determine their nature. **2**

(ii) Sketch the function for $-3 \leq x \leq 2$, showing all stationary points. **2**

(iii) What is the global minimum of the function for the domain $-3 \leq x \leq 2$? **2**

(c) Huwi borrowed \$20,000 from a bank at an interest rate of 10% per annum, where interest is charged quarterly. He makes repayments of \$$P$ to the bank at the end of each quarter, where the amount owing on the loan after n payments is B_n.

(i) Show that $B_2 = 20000 \times 1.025^2 - P \times 1.025 - P$. **1**

(ii) Show that $B_n = 20000 \times 1.025^n - 40P \times (1.025^n - 1)$. **2**

(iii) If the loan was fully repaid after 7 years, what would the value of P be? Express your answer to the nearest dollar. **2**

(iv) If Huwi were to pay \$900 per quarter in repayments, how long would it take (to the nearest quarter) for him to fully repay the loan? **2**

End of Question 15.

Question 16 (15 marks) Use a NEW page on your OWN PAPER.

(a) Differentiate with respect to x: $y = 5^x + 5x$ **2**

(b) State the coordinates of the focus for the parabola: $y = x^2 + 4x + 2$. **2**

(c) The mass M of an 8-gram radioactive substance decays over time t (in days) according to the formula:

$$M = 8e^{-kt}$$

where k is a positive constant.

(i) Show that M satisfies the differential equation $\dfrac{dM}{dt} = -kM$. **1**

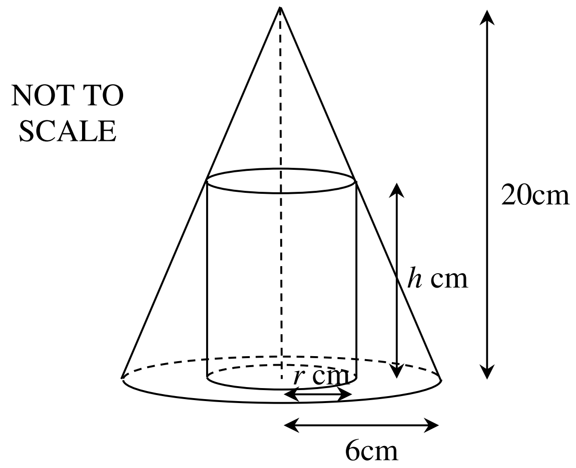
(ii) If the radioactive substance loses 2.4 grams after 12 days,

(α) Find the value of k rounding to two significant figures. **2**

(β) Using (α), find the 'half-life' of the radioactive substance. i.e. the time it takes for the substance to lose half its mass (nearest day). **2**

Question 16 continues on the next page.

(d)



A cylinder of radius r cm and height h cm is inscribed in a cone with base radius 6cm and height 20cm, as shown in the diagram.

(i) Show that the volume, V, of the cylinder is given by $V = \frac{10}{3}\pi r^2(6-r)$. **3**

(ii) Hence, or otherwise, find the values of r and h such that the cylinder has a maximum volume. **3**

End of paper.