Student details

Name: _____

Mark: _____

2022

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 100

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 7 – 14

90 marks

- Attempt Questions 11 – 31
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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1 What is the limiting sum of the series $12 + 4 + \frac{4}{3} + \ldots$?

(A) 9

(B) 14

(C) 16

(D) 18

2 What are the solutions for x in the equation $x^2 - x - 6 < 0$?

(A) $x \in (-1, 6)$

(B) $x \in (-\infty, -1) \cup (6, \infty)$

(C) $x \in (-\infty, -2) \cup (3, \infty)$

(D) $x \in (-2, 3)$

3 Which of the following is a solution for x in the equation: $2\sin x - \sqrt{3} = 0$

(A) $x = 0$

(B) $x = 0.5$

(C) $x = \frac{\pi}{3}$

(D) $x = \frac{\pi}{6}$

4 The following table shows the function values for the $y = f(x)$:

x	3	3.5	4	4.5	5
$f(x)$	4.2	2.5	3.9	6.7	7.4

Using the Trapezoidal rule, which of the following is an approximation for $\int_3^5 f(x)\,dx$ using five function values?

(A) 9.45

(B) 18.9

(C) 20.82

(D) 26.025

5 What is the derivative of $\log_e\left(x^5 + 4\right)$?

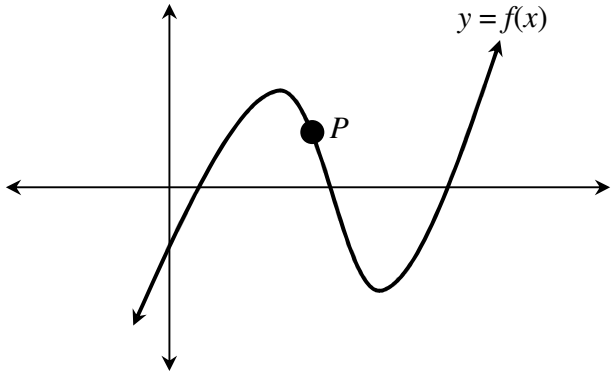
(A) $\dfrac{1}{x^5 + 4}$

(B) $\dfrac{5x^4}{x^5 + 4}$

(C) $\dfrac{5x^4}{\log_e\left(x^5 + 4\right)}$

(D) $\dfrac{1}{\log_e\left(x^5 + 4\right)}$

6 The following diagram shows a sketch of $y = f(x)$, where the point P lies on the curve.



What statement is true about the point P?

(A) $f'(x) > 0$ and $f''(x) > 0$

(B) $f'(x) < 0$ and $f''(x) > 0$

(C) $f'(x) > 0$ and $f''(x) < 0$

(D) $f'(x) < 0$ and $f''(x) < 0$

7 A particle is moving along the x-axis in a straight line. After t seconds, the particle's velocity and acceleration is -6 ms^{-1} and 2 ms^{-2} respectively.

Which statement best describes the motion of the particle after t seconds?

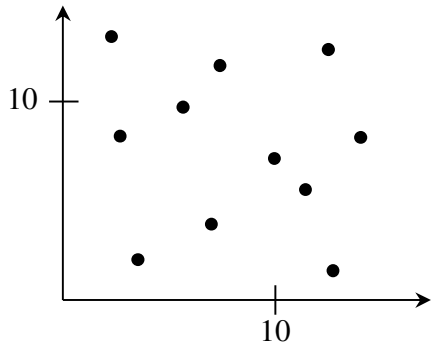
(A) The particle is moving to the left and is slowing down.

(B) The particle is moving to the right and is slowing down.

(C) The particle is moving to the left and is speeding up.

(D) The particle is moving to the right and is speeding up.

8 The follow diagram shows a scatter plot for a set of data.



Which of the following values is the most appropriate Pearson's correlation coefficient for this set of data?

(A) –0.9

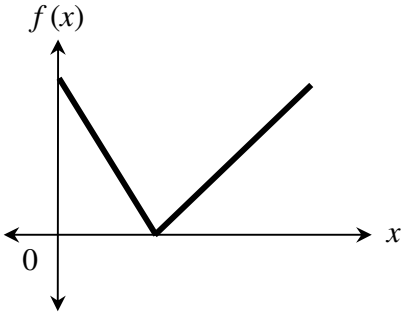
(B) –0.1

(C) 0.5

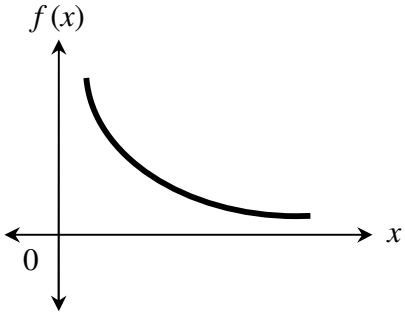
(D) 0.9

9 Which of the following graphs CANNOT represent a probability density function?

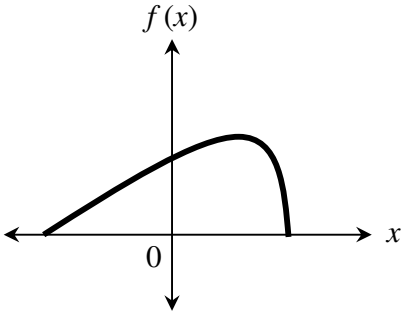
(A)



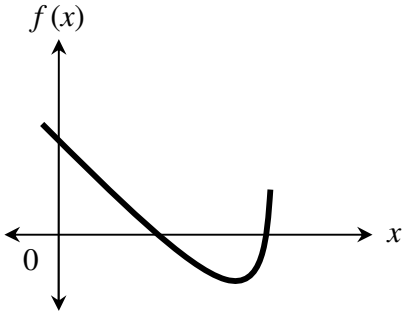
(B)



(C)



(D)



10 The function $y = (x - 5)^2(4 - x)$ undergoes the following transformations:

- reflection about the y-axis,
- vertical dilation by a factor of $\frac{1}{4}$, and
- horizontal dilation by a factor of 2.

Which of the following represents the equation of the transformed function?

(A) $y = -4(2x - 5)^2(4 - 2x)$

(B) $y = -\frac{1}{4}(2x - 5)^2(4 - 2x)$

(C) $y = -\frac{1}{2}\left(\frac{x}{2} - 10\right)^2\left(8 - \frac{x}{2}\right)$

(D) $y = \frac{1}{32}(x + 10)^2(8 + x)$

Section II

90 marks
Attempt Questions 11–31

In Questions 11–31, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Solve for x: $|2x-3| = |x+5|$. **2**

Question 12

Prove the identity: $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{\sin\theta}{1+\cos\theta} = 2\cot\theta$. **2**

Question 13

Differentiate the following with respect to x:

(a) $y = \left(x^3 - 5\right)^4$. **1**

(b) $y = \dfrac{x^3+6}{5-2x^2}$. **2**

(c) $y = x\sin 2x$. **2**

(d) $y = \log_e\left(e^{3x} - 1\right)$. **2**

Question 14

Solve for x: $\sqrt{x} + \sqrt{x+1} = 5$. **2**

Question 15

An arithmetic progression has 28 as the third term and –125 as the twelfth term.

(a) Find the value of the first term and the common difference. **2**

(b) Find the value of the 18^{th} term. **1**

Question 16

Find:

(a) $\int 3\sin 2x\, dx$. **1**

(b) $\int \frac{4x^3 + 2x^2 + 3x + 8}{x}\, dx$. **2**

(c) $\int_0^2 \frac{4x}{x^2 + 3}\, dx$. **2**

Question 17

If $\tan\theta = \frac{9}{5}$ and $\cos\theta < 0$, find the exact value of $\text{cosec}\,\theta$. **2**

Question 18

Consider the function $f(x) = \cos x$. **2**

The function $g(x)$ is a transformation of $f(x)$ with an amplitude of 3 and period of $\frac{\pi}{2}$.

Find the equation of $g(x)$.

Question 19

If $\log_e 2 + \log_e (x + 4) = 2\log_e x$, find the value of x. **2**

Question 20

The probability distribution of a discrete random variable X is summarised in the following table:

x	2	4	5	8	9
$\mathbf{P}(X = x)$	0.1	$a + 4b$	0.05	$5b - a$	0.4

where a and b are constants and $P(X \leq 4) = 0.4$.

(a) Find the value of a and b. **2**

Using part (a),

(b) Find the expected value of X. **1**

(c) Find the variance of X. **1**

Question 21

Find the equation of the normal to the curve $y = 2\sin x + 1$ at the point $(\pi, 1)$. **3**

Question 22

The electrical charge Q retained by a capacitor t minutes after charging is giving by:

$$Q = Q_0 e^{-kt}$$

where Q_0 and k are positive constants.

The charge after 20 minutes is one half of the initial charge.

(a) Show that $k = \frac{1}{20}\log_e 2$ **2**

(b) How long will it be before only one tenth of the original charge is retained? Round your solution to the nearest minute. **2**

Question 23

A particle moves along a straight line where its velocity v m/s after time t seconds is given by the formula:

$$v = 3t^2 - 6t - 24.$$

Initially, the particle is located 3m to the left of the origin.

(a) When does the particle come to rest? **1**

(b) In terms of t, find an expression for the particle's acceleration a. **1**

(c) Find the particle's minimum velocity? **2**

(d) In terms of t, find an expression for the particle's displacement x. **2**

Question 24

The following table shows the present value of a $1 annuity across various interest rates and periods. **2**

Present Value of a $1 Annuity					
End of Year	**Interest Rate (per period)**				
	3.5%	**4.0%**	**4.5%**	**5.0%**	**5.5%**
1	0.9662	0.9615	0.9569	0.9524	0.9479
2	1.8997	1.8861	1.8727	1.8594	1.8463
3	2.8016	2.7751	2.7490	2.7232	2.6979
4	3.6731	3.6299	3.5875	3.5460	3.5052
5	4.5151	4.4518	4.3900	4.3295	4.2703
6	5.3286	5.2421	5.1579	5.0757	4.9955
7	6.1145	6.0021	5.8927	5.7864	5.6830
8	6.8740	6.7327	6.5959	6.4632	6.3346
9	7.6077	7.4353	7.2688	7.1078	6.9522
10	8.3166	8.1109	7.9127	7.7217	7.5376

Ham invested in an annuity product that involved him paying a large upfront payment to an institution and then receiving equal payments of $4500 each year for seven years. The annuity product was quoted to pay interest at 5.5% p.a.

Using the table of values, find how much Ham paid for the annuity at the start of the seven years.

Question 25

A school held a raffle where 100 tickets were sold in total. The raffle had two prizes to be won where the first ticket drawn for the first prize will be discard before the second ticket is drawn.

Pan bought a number of tickets where his probability of winning both prizes was $\frac{1}{75}$.

(a) Find the number of tickets that Pan bought. **2**

(b) Hence, or otherwise, find the probability that Pan wins at least one prize. **1**

Question 26

(a) Show that $\frac{d}{dx}\left(\log_e\left(\frac{5+x}{5-x}\right)\right) = \frac{10}{25-x^2}$. **3**

(b) Hence, or otherwise, find $\int \frac{dx}{25-x^2}$. **1**

Question 27

Kang takes out a home loan of $800,000 from a bank at a reducible interest rate on of 2.4% per annum, compounded monthly, to be repaid over 30 years. Interest will be calculated at the end of each month, just before the monthly repayment M is made.

The amount owning on the loan after n periods is denoted by A_n.

(a) Show that the amount owing after the n^{th} repayment is made is: **3**

$$A_n = 800000 \times 1.002^n - 500M\left(1.002^n - 1\right).$$

(b) Hence, or otherwise, show that the monthly repayment M is $3119.53. **2**

(c) After 12 months, the amount owning on the loan is $781,564 [**DO NOT PROVE THIS**]. At this point the bank increases the reducible interest rate to 3% per annum. **2**

If Kang decides that he can afford to continue to make repayments of M, using the rounded result in part (b), how many more monthly repayments will be made from this point?

Question 28

The following table summarises the recent numeracy test results (out of 50 marks) for a group of Year 6 students.

Age in months (*A*)	**Result (*R*)**
135	38
142	41
138	32
134	33
140	48
135	35
136	39
138	40
141	49
139	30
137	35
135	45

By using the table above, and rounding your solution to two decimal places where required:

(a) State the independent variable. **1**

(b) Determine the value of Pearson's correlation coefficient (r). **1**

(c) Find the 'line of best fit', stating your solution as $R = \square + \square \times A$. **2**

(d) Using your solution to part (c), estimate a student's result if they were 138 months old, rounding your solution to the nearest mark. **1**

Question 29

A continuous random variable X has a probability density function:

$$f(x) = \begin{cases} \dfrac{x^2}{3} & 0 \leq x \leq \sqrt[3]{9} \\ 0 & \text{otherwise} \end{cases}$$

Rounding to three decimal places,

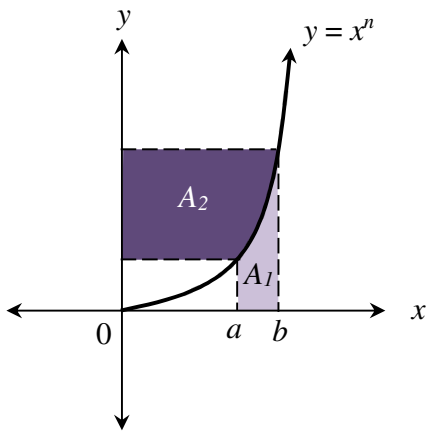
(a) Find the expected value of X. **2**

(b) Find the median value of X. **2**

(c) Find the interquartile range of X. **3**

Question 30

The diagram shows the graph of $y = x^n$, where $x \geq 0$ for $n > 1$. **3**



If the area of A_2 is three times larger than A_1, find the value of n.

Question 31

Consider the function $y = \left(x^2 + 1\right)e^{-x}$.

(a) Find any stationary point(s) and determine their nature. **3**

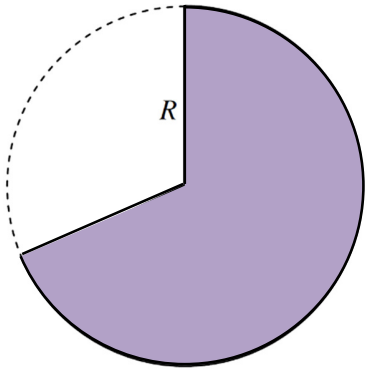
(b) Find any points of inflection. **2**

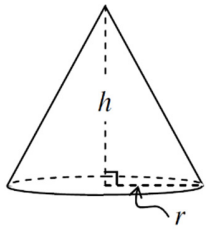
(c) State the equation of the horizontal asymptote. **1**

(d) Hence, or otherwise, sketch the curve $y = \left(x^2 + 1\right)e^{-x}$ on a number plane, showing all key features. **2**

Question 32

A circular 100cm^2 metallic disc with radius of R cm had a sector cut out to form a right cone with height h cm and radius r cm, as shown in the diagram below.





(a) Show that the volume, V, of the cone is given by $V = \dfrac{r^2\sqrt{100\pi - \pi^2 r^2}}{3}$ cm^3. **2**

(b) Show that the volume of the cone is maximised when $r = \sqrt{\dfrac{200}{3\pi}}$. **3**

(c) Given the area of the sector used to make the cone is $A = \pi Rr$ show that the angle in this sector, which gives a maximum volume for the cone, is $\theta = \dfrac{2\pi\sqrt{6}}{3}$ radians. **2**

End of paper.