Student details

Name: ______________________________

Mark: ______________________________

2022

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 6 – 11

60 marks

- Attempt Questions 11 – 28
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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1 Which of the following is equivalent to $x^2 - 5x + 6$?

(A) $(x + 2)(x - 3)$

(B) $(x - 2)(x - 3)$

(C) $(x - 1)(x - 6)$

(D) $(x - 1)(x + 6)$

2 Which of the following represents the Cartesian equation of $(2\cos\theta - 1,\ 2\sin\theta + 3)$?

(A) $x^2 + y^2 = 4$

(B) $x^2 + y^2 = 1$

(C) $(x + 1)^2 + (y - 3)^2 = 4$

(D) $(x - 1)^2 + (y + 3)^2 = 4$

3 Which of the following are the solutions for $x \in [0, 2\pi]$: $\sqrt{2}\sin x = -1$

(A) $x = \frac{\pi}{4}$

(B) $x = \frac{\pi}{4}, \frac{5\pi}{4}$

(C) $x = \frac{5\pi}{4}, \frac{7\pi}{4}$

(D) $x = \frac{\pi}{4}, \frac{3\pi}{4}$

4 If eight students were seated around a round table, how many unique arrangements are possible if three particular students were to be seated adjacent to each other?

(A) 8!3!

(B) 7!3!

(C) 6!3!

(D) 5!3!

5 Which of the following can be a solution to the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

(A) $y = \sin x$

(B) $y = e^x$

(C) $y = \ln(x)$

(D) $y = \sqrt{x^2 - 4}$

6 Which of the following equals to the coefficient of x^7 in the expansion of $\left(2x^2 - \frac{1}{3x}\right)^8$?

(A) $-\frac{448}{243}$

(B) $\frac{1120}{81}$

(C) $-\frac{1792}{27}$

(D) $\frac{1}{6561}$

7 Fred recently started a new job that required him to catch the 7:25am bus every morning. He noted that the bus usually comes no time however there has been days where it was late by a few minutes. Over a period of 80 days, Fred noted that the bus was late on twelve occasions.

Over this 80-day period, what is the standard deviation of times that the bus was late?

(A) 8.944

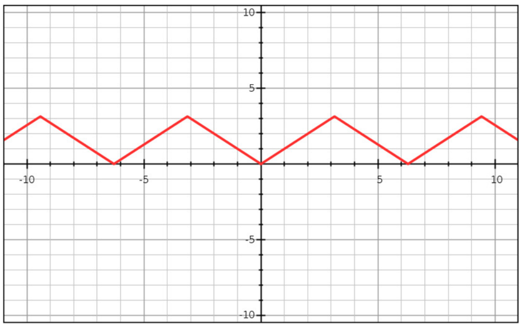
(B) 10.2

(C) 3.194

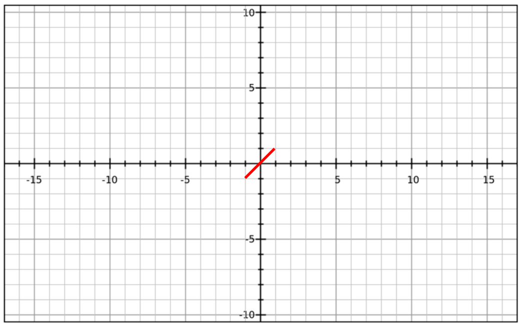
(D) 12

8 Which of the following graphs best represents $y = \cos\left(\cos^{-1} x\right)$?

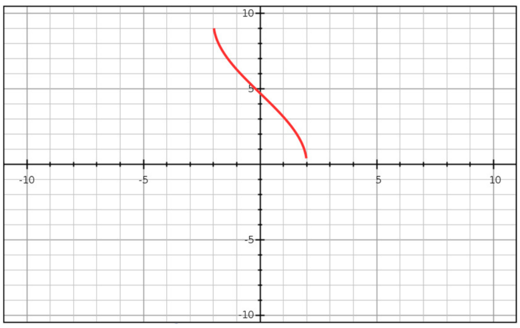
(A)



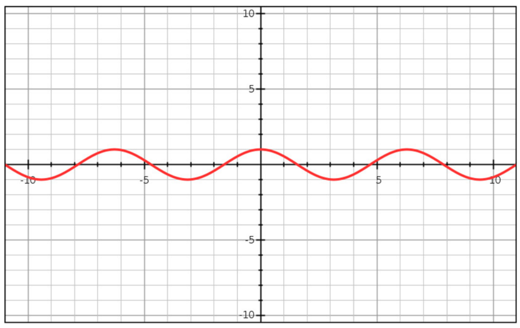
(B)



(C)



(D)



9 When a polynomial $P(x)$ is divided by $(x-1)$, the remainder is 8. When $P(x)$ is divided by $(x+4)$, the remainder is -7.

What is the reminder when $P(x)$ is divided by $(x-1)(x+4)$?

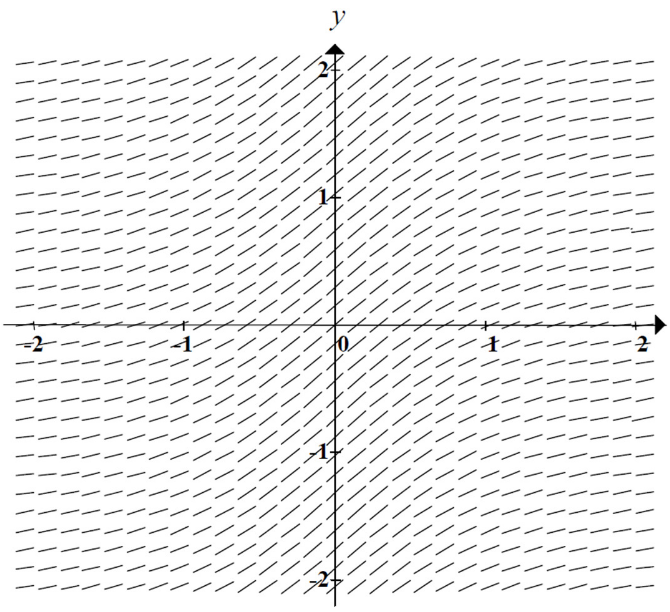
(A) Remainder is $3x+5$

(B) Remainder is $-3x-5$

(C) Remainder is $2x-1$

(D) Remainder is $-2x+1$

10 Which of the following could be the differential equation represented by the slope field below?



(A) $\dfrac{dy}{dx} = \dfrac{1}{|1+x+y|}$

(B) $\dfrac{dy}{dx} = \tan^{-1} x$

(C) $\dfrac{dy}{dx} = |1+x|$

(D) $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$

Section II

60 marks
Attempt Questions 11–28

In Questions 11–28, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Solve for x, expressing you solution in set notation: $\dfrac{2x-1}{4-5x} \le 2$. **2**

Question 12

Find $\displaystyle\int \frac{1}{\sqrt{25-4x^2}}\,dx$. **2**

Question 13

The coordinates A, B and C are represented by the position vectors $\underset{\sim}{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$ **2**
and $\underset{\sim}{c} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$. Find the size of the acute angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$, rounding your solution to the nearest degree.

Question 14

By using the substitution $t = \tan\left(\dfrac{x}{2}\right)$, prove the following identity: **3**

$$\frac{1+\sin x-\cos x}{1+\sin x+\cos x} = \tan\left(\frac{x}{2}\right).$$

Question 15

Find the exact value of $\sin\left(2\tan^{-1}\frac{2}{5}\right)$.   **3**

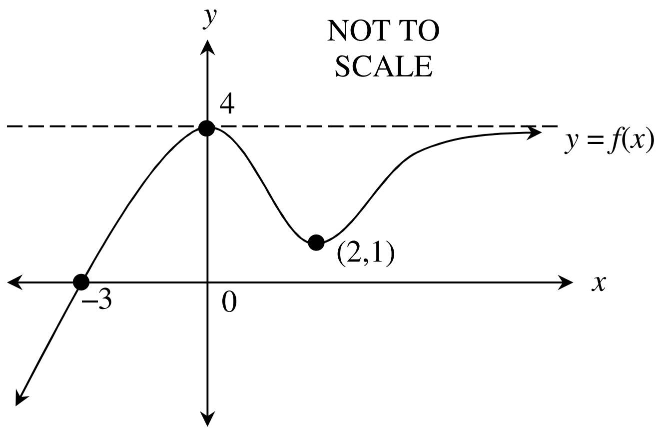
Question 16

A multiple-choice exam had 12 questions, each with choices of A, B, C, D and E.   **2**

If a student randomly guessed all his solutions, what is the probability that they get 75% for exam? Round your solution to the nearest three significant figures.

Question 17

The diagram shows the graph of a function $f(x)$.



Sketch the following curves on separate diagrams:

(a) $y = f\left(|x|\right)$ **1**

(b) $y = \dfrac{1}{f\left(x\right)}$ **2**

(c) $y^2 = f(x)$ **2**

Question 18

The polynomial $P(x) = x^3 - 2x^2 - 4x - 7$ has roots α, β and γ.

(a) Find the value of $\alpha + \beta + \gamma$. **1**

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. **1**

(c) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. **2**

Question 19

Solve for x: $\sin x + \sqrt{3}\cos x = 1$ for $x \in [0, 2\pi]$. **3**

Question 20

Consider the functions $f(x) = x(x+1)$ and $g(x) = x^2 - 8x + 12$.

(a) Find the value of $f(f(1))$. **1**

(b) Draw a neat sketch of the graph $y = g(f(x))$, labelling all key features. **3**

Question 21

Prove by mathematical induction for $n \in \mathbb{Z}^+$: **3**

$$1^2.2 + 2^2.3 + 3^2.4 + ... + n^2(n+1) = \frac{1}{12}n(n+1)(n+2)(3n+1).$$

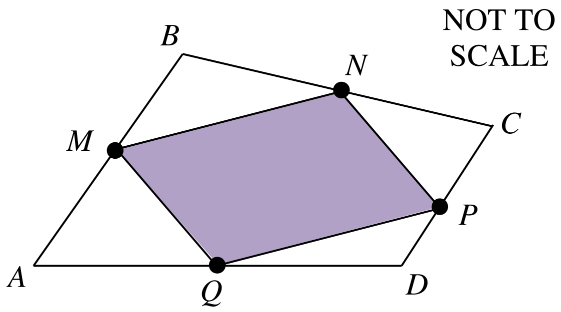
Question 22

How many unique four-letter arrangements are possible using the letters in the word "SCANNING"? **3**

Question 23

Varignon's theorem states that the figure formed by joining the midpoints of all sides of any quadrilateral is a parallelogram. **3**

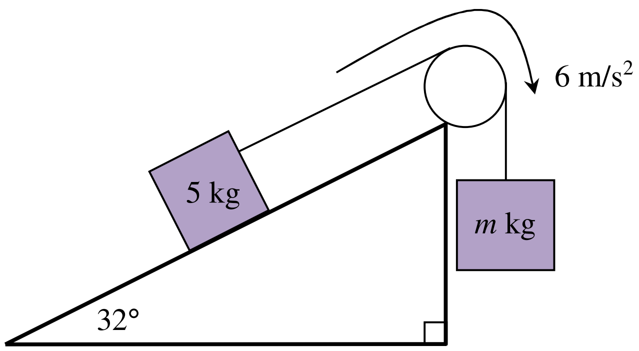
Consider a quadrilateral *ABCD*, where the points *M*, *N*, *P* and *Q* are the midpoints of the sides *AB*, *BC*, *CD* and *DA* respectively, as shown in the diagram below.



Using vectors, prove Varignon's theorem for the quadrilateral *ABCD* (i.e. prove that *MNPQ* is a parallelogram).

Question 24

A 5 kilogram object on an inclined plane was connected to a free hanging object of mass *m* kilograms via a light inextensible string in a pulley system, as shown in the diagram below:



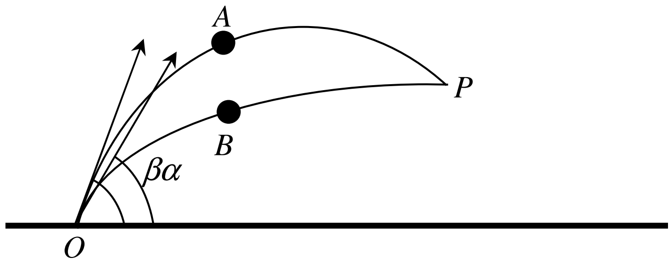
The system accelerated such that the 5 kilogram mass moved upwards at a rate of 6 m/s^2. Assuming gravity of 9.8 m/s^2,

(a) Find the amount of tension in the string. **2**

(b) Find the value of *m*, rounding your solution to one decimal place. **2**

Question 25

An object (A) was projected from the point O on the ground with initial velocity of u m/s at an angle of α to the horizontal. After T seconds, a second object (B) was projected from point O with the same initial velocity as A at an angle of β to the horizontal. The objects collide in the air at point P, as shown in the diagram below.



Assuming gravity is g m/s^2, the equation of the path of that object A travels is given by the following:

$$y = -\frac{gx^2}{2u^2}\sec^2\alpha + x\tan\alpha \qquad \textbf{(DO NOT PROVE THIS)}$$

(a) Write down the equation of the path that object B travels to. **1**

(b) Show that the horizontal distance travelled by both objects when they collide at point P is: **2**

$$x = \frac{2u^2\cos\alpha\cos\beta}{g\sin(\alpha+\beta)}$$

The horizontal displacement of object A after t seconds is given by: $x_A = Vt\cos\alpha$ (**DO NOT PROVE THIS**).

(c) Write down the equation for the horizontal displacement of object B x_B after t seconds. **1**

(d) Show that, for the collision to take place, the value of T is given by: **2**

$$T = \frac{2u(\cos\beta - \cos\alpha)}{g\sin(\alpha+\beta)}.$$

Question 26

Solve for θ for $0 \leq \theta \leq 2\pi$: $\sin\theta - \sin 3\theta + \sin 5\theta = 0$. **3**

Question 27

In a laboratory, an experiment was conducted on a new strain of the *H-Lix* virus. The researchers started their experiment with 2500 virus cells, where the number of virus cells (P) fluctuated over time (t hours) according to the differential equation:

$$\frac{dP}{dt} = \frac{1}{2000} P\left(10000 - P\right)\cos t$$

(a) Show that: $\dfrac{1}{P\left(10000 - P\right)} = \dfrac{1}{10000}\left[\dfrac{1}{P} + \dfrac{1}{10000 - P}\right]$. **1**

(b) Show that the solution to the differential equation is: **3**

$$P = \frac{10000}{1 + 3e^{-5\sin t}}, \text{ where } t \geq 0.$$

(c) Find the range at which the population of virus cells fluctuate between, rounding your solution to the nearest whole number. **1**

Question 28

The equation $ax^2 + bx + c = 0$ has roots $x = \tan\alpha$ and $x = \tan\beta$ where $0 < \alpha < \beta$. **3**

Find an expression for $\tan(\beta - \alpha)$ in terms of a, b and c, expressing your solution in simplest form.

End of paper.