NEXT EDUCATION

2017

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used

Total marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 14

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

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1 What is $\log_e(159)$ to 3 significant figures?

(A) 5.1

(B) 5.07

(C) 5.069

(D) 5.0689

2 Which of the following is a solution for x in the equation: $\sqrt{3}\tan x - 1 = 0$

(A) $x = 0$

(B) $x = \frac{\pi}{6}$

(C) $x = \frac{\pi}{3}$

(D) $x = \frac{2\pi}{3}$

3 A factory had 56 employees, of which 35 spoke English, 29 spoke Italian, and some spoke both languages. How many employees spoke both English and Italian?

(A) 27

(B) 21

(C) 9

(D) 8

4 What is the perpendicular distance between the point (2,–1) and the line $y = 4x - 5$?

(A) $\frac{2}{\sqrt{5}}$ units

(B) $\frac{2}{\sqrt{17}}$ units

(C) $\frac{4}{\sqrt{5}}$ units

(D) $\frac{4}{\sqrt{17}}$ units

5 The equation $x^2 + 4x - 1 = 0$ has roots $x = \alpha$ and $x = \beta$. What is the value of $\alpha^2 + \beta^2$?

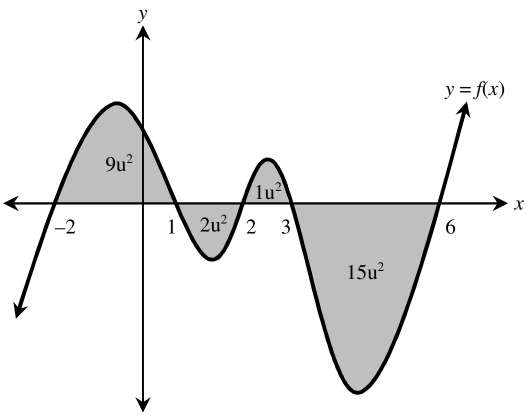
(A) 5

(B) 14

(C) 18

(D) –7

6 The following diagram shows the graph $y = f(x)$, with the area of certain sections labelled.



Using the graph, evaluate:

$$\int_{-2}^{3} f(x)\,dx + \int_{1}^{6} f(x)\,dx$$

(A) −7

(B) −8

(C) 7

(D) 8

7 A sector of a 2m radius circle had an arc length of 4m. Find the area of this sector.

(A) 2m^2

(B) 4m^2

(C) $2\pi\text{m}^2$

(D) $4\pi\text{m}^2$

8 What are the solutions to the equation: $e^{6x} - 5e^{3x} + 4 = 0$?

(A) $x = 0, \dfrac{\ln 4}{3}$

(B) $x = 0, \dfrac{\ln 3}{4}$

(C) $x = 0, \dfrac{\ln 5}{3}$

(D) $x = 0, \dfrac{\ln 4}{5}$

9 The following table shows the values for the function $y = f(x)$:

x	2	3	4	5	6
y	1.3	1.8	2.9	3.8	2.8

Using Simpson's rule, which of the following is an approximation for $\int_2^6 f(x)\,dx$?

(A) 8.97

(B) 10.77

(C) 13.45

(D) 16.15

10 A particle is moving along the x-axis in a straight line. After t seconds, the particle's velocity and acceleration is $\pi\,\text{ms}^{-1}$ and $-2\pi\,\text{ms}^{-2}$ respectively.

Which statement best describes the motion of the particle after t seconds?

(A) The particle is moving to the left and is slowing down.

(B) The particle is moving to the right and is slowing down.

(C) The particle is moving to the left and is speeding up.

(D) The particle is moving to the right and is speeding up.

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question on a NEW page on your OWN PAPER.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW page on your OWN PAPER.

(a) Factorise: $8m^2 - 98$. **2**

(b) Evaluate $\lim\limits_{x \to \infty} \dfrac{4x - x^2}{3x^2 - 5x + 7}$. **1**

(c) Differentiate the following with respect to x:

(i) $e^{\tan x}$. **2**

(ii) $\dfrac{x^3}{\sqrt{1 - x^2}}$. **2**

(d) Find the exact value of $\log_2\left(\dfrac{1}{\sqrt{2}}\right)$. **2**

(e) Solve for x: $\dfrac{9 - x}{2x + 3} \geq 0$. **2**

(f) If $\tan\theta = -\dfrac{3}{5}$ and $\cos\theta < 0$, find the exact value of $\operatorname{cosec}\theta$. **2**

(g) Find the limiting sum of the series: $2 - \dfrac{6}{5} + \dfrac{18}{25} - \dfrac{54}{125} + \dots$ **2**

Question 12 (15 marks) Use a NEW page on your OWN PAPER.

(a) For the parabola $(x - 3)^2 = 2(y + 1)$,

(i) Find the coordinates of the vertex. **1**

(ii) Find the coordinates of the focus. **1**

(iii) Find the equation of the directrix. **1**

(b) Find:

(i) $\int \frac{6x^2}{4 + x^3}\, dx$. **1**

(ii) $\int \sin(2017x)\, dx$. **1**

(iii) $\int \frac{4}{x^3} + \frac{e^{3x}}{4}\, dx$. **2**

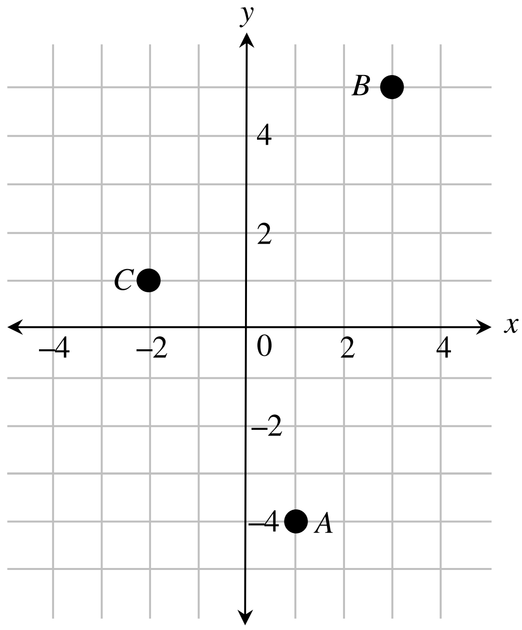
(c) Prove the identity: $\frac{(1 + \tan^2 x)\cot x}{\operatorname{cosec}^2 x} = \tan x$. **2**

(d) Find the equation of the tangent to the curve $y = \ln\left(\frac{2x - 1}{x + 1}\right)$ at the point where $x = 2$. **3**

(e) Solve for x: $3^{2(x + 1)} - 10(3^x) + 1 = 0$. **3**

Question 13 (15 marks) Use a NEW page on your OWN PAPER.

(a) Points $A(1,-4)$, $B(3,5)$ and $C(-2,1)$ lie on a Cartesian plane, as shown in the diagram below.



(i) Find the length of the interval AB. **1**

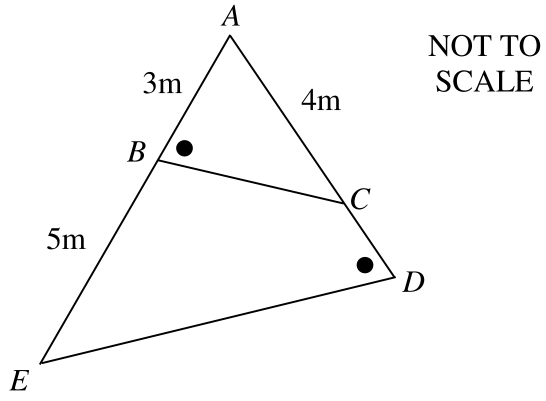
(ii) Find the equation of a straight line passing through A and B, leaving your answer in general form. **2**

(iii) Find the perpendicular distance between the point C and the line passing through A and B. **2**

(iv) Hence, or otherwise, find the area of ΔABC, leaving your answer in exact form. **1**

(b) Solve for x: $\left(\log_2 x^3\right)\left(\log_2 x\right) = 12.$ **2**

(c)



In the diagram, $AB = 3\text{m}$, $BE = 5\text{m}$, and $AC = 4\text{m}$.

Given that $\angle ABC = \angle ADE$,

(i) Prove that the triangles ABC and ADE are similar. **2**

(ii) Hence, find the length CD giving reasons. **2**

(d) A colony of insects was observed as part of a study, where the rate at which the population (P) increases at is given by the equation:

$$\frac{dP}{dt} = kP.$$

where k is a positive constant.

(i) Verify that $P = Ae^{kt}$ is a solution to the differential equation where A is the initial population of insects observed. **1**

(ii) If the initial population of insects was 680 and doubles after 30 days, find the value of k to three decimal places. **2**

Question 14 (15 marks) Use a NEW page on your OWN PAPER.

(a) A particle moves along a straight line where its velocity v ms^{-1} after time t seconds is given by the formula:

$$v = 3t^2 - 24t + 36.$$

Initially, the particle is 2 metres to the right of the origin O.

(i) In terms of t, find an expression for the particle's displacement x in metres. **1**

(ii) Find when and where the particle is at rest. **2**

(iii) Find the total distance travelled by the particle over the first 4 seconds. **2**

(b) (i) Solve for x, where $0 \le x \le 2\pi$: $4\sin(2x + \pi) + 2 = 0$. **3**
Leave your solution in exact form.

(ii) Draw a neat sketch of $y = 4\sin(2x + \pi) + 2$, where $0 \le x \le 2\pi$, showing all intercepts. **2**

(c) (i) Differentiate with respect to x: $\ln(\ln x)$. **2**

(ii) Hence, or otherwise, find the exact value of: $\int_e^{e^2} \frac{dx}{x \ln x}$. **3**

Question 15 (15 marks) Use a NEW page on your OWN PAPER.

(a) A soccer team has eight more games to play till the season finishes. For each game, the team can either win, lose or draw. If the probability of a win, loss and draw are identical, find the probability that the team wins at least one of the eight games. **2**

(b) For the curve $y = \dfrac{4x}{x^2 + 1}$,

(i) Find the y-intercept. **1**

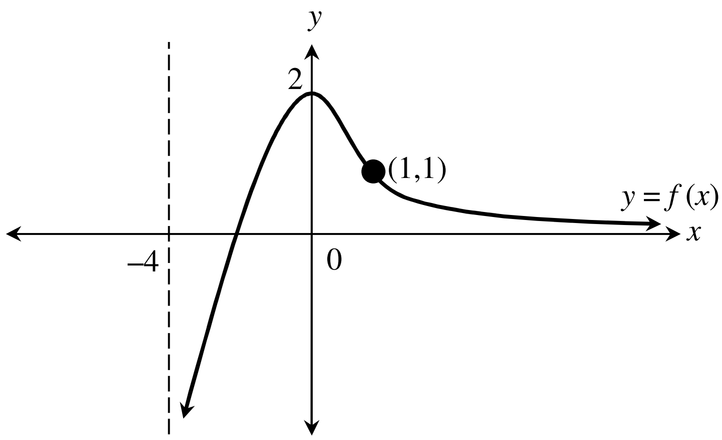
(ii) Find the stationary points and determine their nature. **4**

(iii) Explain why the curve does not have any vertical asymptotes. **1**

(iv) Using limits, find the horizontal asymptote. **1**

(v) Hence, or otherwise, sketch the curve on a number plane, showing all key features. **2**

(c) The following diagram shows the graph $y = f(x)$:



The graph has a maximum turning point at (0,2) and a point of inflexion at (1,1).

(i) Draw a neat sketch of $y = f'(x)$, showing all key features. **2**

(ii) Draw a neat sketch of $y = f''(x)$, showing all key features. **2**

Question 16 (15 marks) Use a NEW page on your OWN PAPER.

(a) Consider the following equation: **2**

$$(a^2 + b^2)x^2 + 2b(a + c)x + (b^2 + c^2) = 0$$

Find the conditions for a, b and c such that the equation has real roots.

(b) To purchase an apartment, Klap took out a loan of \$600,000 from a bank at a reducible interest rate of 3% per annum (compounding monthly). He makes monthly repayments of \$$B$ to the bank at the end of each month, where the amount owing on the loan after n payments is A_n.

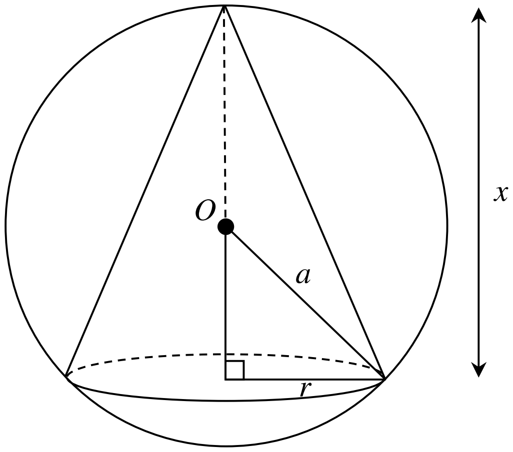
(i) Show that $A_2 = 600000 \times 1.0025^2 - B \times 1.0025 - B$. **1**

(ii) Show that $A_n = 600000 \times 1.0025^n - 400B \times (1.0025^n - 1)$. **2**

(iii) Hence, of otherwise, find the value of B if the loan is repaid over 30 years (360 months). **1**

(iv) Halfway through the loan duration (after 180 months), Klap decided to double his repayments to \$$2B$. How much faster will he repay the loan (nearest month). **3**

(c)



A cone is inscribed in a sphere of radius a, centred at O. The height of the cone is x and the radius of the base is r, as shown in the diagram.

(i) Show that the volume, V, of the cone is given by $V = \frac{1}{3}\pi\left(2ax^2 - x^3\right)$. **3**

(ii) Find the value of x for which the volume of the cone is a maximum. **3**

End of paper.