NEXT EDUCATION

2020

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen.
- Board-approved calculators may be used.
- All answers should be written on this examination paper.
- There is some extra writing space at the end of this paper.

Total marks – 70

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 7 – 22

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Circle the best solution below for Questions 1 – 10

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1 What is the solution to x: $\dfrac{3}{x-2} \leq -1$?

(A) $x \in [-1, 2)$

(B) $x \in [-1, 2]$

(C) $x \in [-3, 2)$

(D) $x \in [-3, 2]$

2 Which of the following is equivalent to $\displaystyle\int \frac{2}{25 + x^2}\, dx$?

(A) $2\sin^{-1}\left(\dfrac{x}{5}\right) + \text{c}$

(B) $\sin^{-1}\left(\dfrac{2x}{5}\right) + \text{c}$

(C) $\dfrac{2}{5}\tan^{-1}\left(\dfrac{x}{5}\right) + \text{c}$

(D) $\tan^{-1}\left(\dfrac{2x}{5}\right) + \text{c}$

3 What is the derivative of $x\sin x$?

(A) $\cos x$

(B) $1 + \cos x$

(C) $\sin x + x\cos x$

(D) $\cos^2 x$

4 How many unique arrangements of the letters in the word "RECOVERY" if the two E's are placed next to each other?

(A) $\frac{7!}{2!}$

(B) $\frac{8!}{2!}$

(C) $\frac{7!}{2!2!}$

(D) $\frac{8!}{2!2!}$

5 If $4\sin x + 3\cos x = R\sin(x + \alpha)$, which of the following is the closest to the value of α in radians?

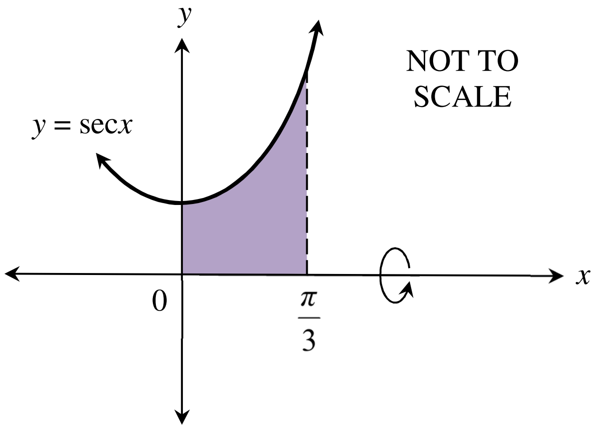
(A) 5

(B) 37

(C) 0.64

(D) 0.92

6 The diagram below shows the region bounded by the curve $y = \sec x$, the line $x = \frac{\pi}{3}$, the x-axis and the y-axis.



What is the exact value of the volume formed when the shaded region is rotated about the x-axis?

(A) 2 units^3

(B) 4π units^3

(C) $\sqrt{3}\pi$ units^3

(D) $2\sqrt{3}\pi$ units^3

7 What is the domain and range of the inverse function $y = 4\sin^{-1}(3x + 5)$?

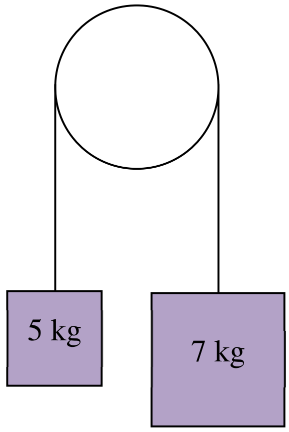
(A) Domain: $x \in [0, 4]$; Range: $y \in [-2\pi, 2\pi]$

(B) Domain: $x \in \left[-\frac{5}{3}, \frac{5}{3}\right]$; Range: $y \in [-4\pi, 4\pi]$

(C) Domain: $x \in \left[-2, -\frac{4}{3}\right]$; Range: $y \in [-2\pi, 2\pi]$

(D) Domain: $x \in [-6, -4]$; Range: $y \in [-3\pi, 3\pi]$

8 Two objects with mass 5 kg and 7 kg are attached via an inextensible string in a smooth pulley system, as shown below:



The objects were held at rest and then released. If T is the tension in the string and the acceleration due to gravity is g m/s^2, the acceleration of the 7 kg mass is:

(A) $\dfrac{T - 5g}{7}$

(B) $\dfrac{T - 5}{5g}$

(C) $\dfrac{7g - T}{7}$

(D) $\dfrac{T - 7}{7g}$

9 When the polynomial $P(x)$ is divided by $(x^2 + x - 2)$, the remainder is $(4x - 3)$. What is the remainder when $P(x)$ is divided by $(x - 1)$?

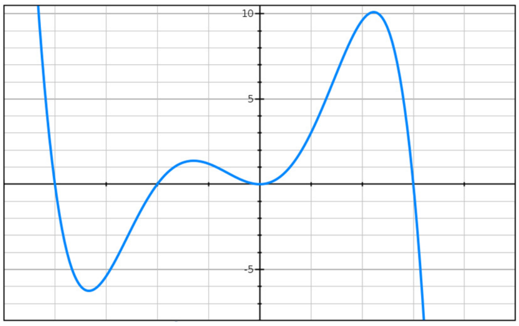
(A) 1

(B) –3

(C) –2

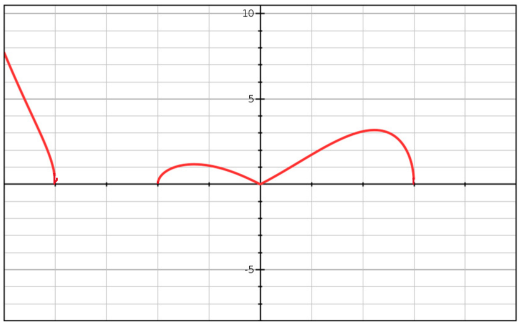
(D) –4

10 The following diagram shows the graph of $y = \dfrac{x^2(x+4)(x+2)(3-x)}{10}$:

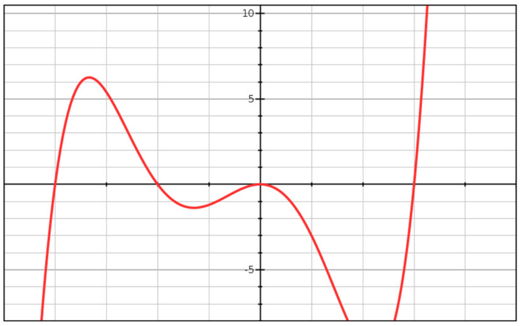


Which of the following graphs best represents $y = \dfrac{\sqrt{x^2(x+4)(x+2)(3-x)}}{\sqrt{10}}$?

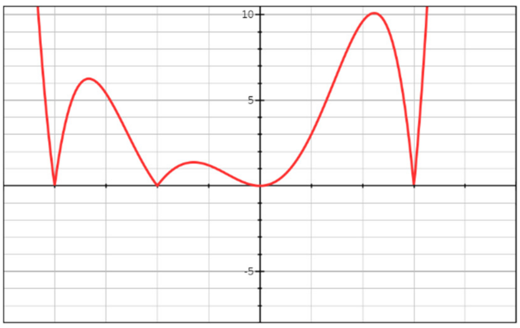
(A)



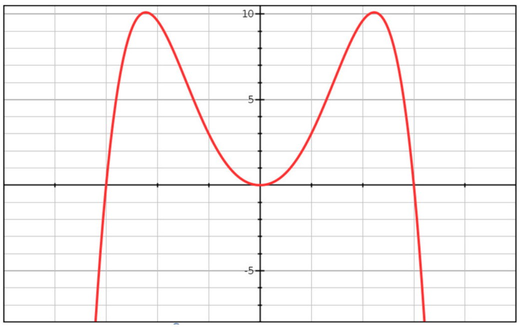
(B)



(C)



(D)



End of Section I.

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

You should include all your solutions and working in the spaces provided in this paper.

Question 11 (15 marks)

(a) Solve for x: $|x-9| = 4x+1$. **2**

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(b) Find the Cartesian equation of the parametric equations: **2**

$$x = 4\cos\theta - 3 \quad \text{and} \quad y = 4\sin\theta + 2.$$

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Question 11 continues on the next page.

(c) Find $\int \sin^2 x \, dx$. **1**

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(d) Use the substitution $u = 1 + x$ to evaluate $\int_0^1 \frac{x}{\sqrt{1+x}} \, dx$. **3**

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(e) Find the exact value of $\sin\left(2\cos^{-1}\frac{3}{5}\right)$. **3**

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Question 11 continues on the next page.

(f) The polynomial $P(x) = x^3 - 4x^2 + 5x - 3$ has roots α, β and γ.

(i) Find the value of $\alpha + \beta + \gamma$. **1**

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(ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. **1**

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(iii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. **2**

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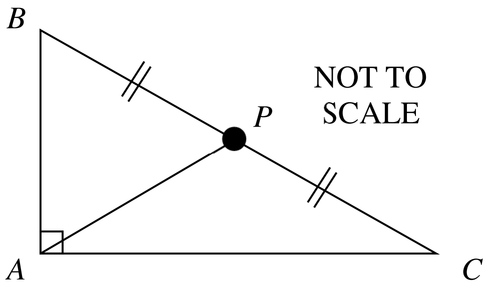
End of Question 11.

Question 12 (15 marks)

(a) Use mathematical induction to prove that $3^{2n+4} - 2^{2n}$ is divisible by 5 for all integers $n \geq 1$. **3**

Question 12 continues on the next page.

(b) ΔABC is a right-angled triangle where P is the midpoint of BC, as shown in the diagram below.



Let $\overrightarrow{BP} = \overrightarrow{PC} = \underset{\sim}{a}$ and $\overrightarrow{AP} = \underset{\sim}{b}$.

(i) Find an expression for $\overrightarrow{BA}$ and $\overrightarrow{AC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

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(ii) Hence, prove that P is equidistant from the three vertices of ΔABC. **2**

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Question 12 continues on the next page.

(c) 10 friends (five men and five women) arrive at a restaurant where they were to be seated for dinner.

If they were seated around a circular table, how many unique arrangements are possible if:

(i) If no restrictions applied? **1**

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(ii) All the women sat next to each other? **1**

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If the only seating arrangement involved two circular tables, one with six seats and another with 4 seats, how many unique arrangements are possible if:

(iii) If no restrictions applied? **1**

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(iv) If a particular man and a particular woman sat together? **2**

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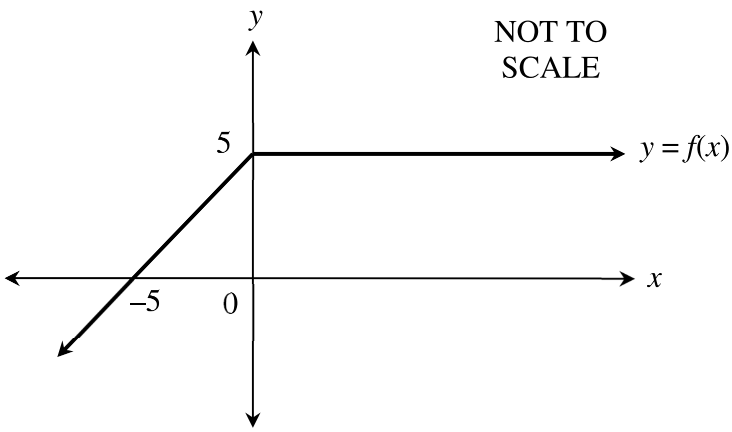
Question 12 continues on the next page.

(d) The diagram below shows the graph of a function $y = f(x)$.



Sketch the following curves on separate diagrams in the space below, labelling all key features.

(i) $y^2 = f\left(x\right)$ **2**

Question 12 continues on the next page.

(ii) $y = \dfrac{1}{f(x)}$ **2**

End of Question 12.

Question 13 (15 marks)

(a) Find the term independent of x in the expansion of $\left(2x^2 + \frac{3}{x}\right)^{12}$. **3**

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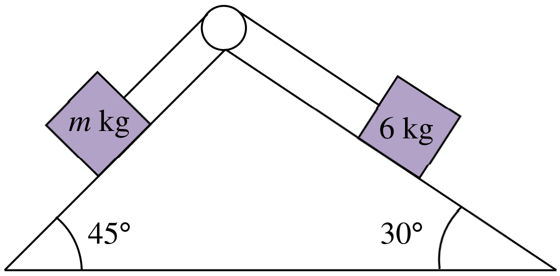
(b) Two objects with mass m kg and 6 kg inclined at angles of 45° and 30° respectively were connected via a light inextensible string in a pulley system, as shown in the diagram below:



If the 6 kg mass is accelerating downwards at 2 m/s^2, and assuming gravity is 9.8 m/s^2,

(i) Find the amount of tension in the string. **2**

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Question 13 continues on the next page.

(ii) Find the value of m, rounding your answer to two decimal places. **2**

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(c) In a recent news report, a new global pandemic has been announced relating to the spread of the contagious virus strain M-13. Pip, one of the 20,000 inhabitants of the small island nation of Nextopia, was the nation's leading statistician and was charged to form a team of researchers to assess, monitor and model the viral infection on the Nextopia's inhabitants. Via a strict testing regime by the government, Pip's team discovered 50 inhabitants had contracted the M-13 virus after disembarking from an international flight. These inhabitants were the first recorded cases on the island where, in the absence of all other data, this recorded date was deemed to be the initial date of the infection for Nextopia.

One of Pip's junior analysts proposed to model the number of infected inhabitants (P) after t days to follow the differential equation: $\frac{dP}{dt} = kP$, where k is a constant.

(i) Provide a reason why this differential equation would not be suitable for modelling the number of infected Nextopia inhabitants. **1**

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Pip proposed that a more suitable approach to model the rate of infection is to use the logistical equation: $\frac{dP}{dt} = kP(20000 - P)$, where k is a constant.

(ii) Show that: $\frac{1}{P(20000-P)} = \frac{1}{20000}\left(\frac{1}{P} + \frac{1}{20000-P}\right)$. **1**

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Question 13 continues on the next page.

(iii) Hence, or otherwise, show using integral calculus that the general solution to the logistical equation is: **3**

$$P = \frac{20000}{1 + Be^{-20000kt}}, \text{ where } B \text{ and } k \text{ are constants.}$$

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Question 13 continues on the next page.

(iv) After 14 days from the initial date of infection, the number of infected inhabitants was found to be 2200. Using this data, find the value of k to three significant figures. **1**

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(v) Using part (iv), estimate the number of infected inhabitants after 28 days after initial date of infection (round your answer to the nearest whole number). **1**

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(vi) In the space below, draw a neat sketch of the number of infected inhabitants over time, showing all key features. **1**

End of Question 13.

Question 14 (15 marks)

(a) A spinner is made up of four sections labelled with the numbers 4, 5, 6 and 7. The probability of a marker arrow landing on each of the sections represents a discrete random variable X and is summarised in the table below:

x	4	5	6	7
P($X = x$)	$\frac{k}{20}$	$\frac{k^2 - 4}{10}$	$\frac{k}{10}$	$\frac{k-2}{20}$

where k is a constant.

(i) Show that $k = 3$. **1**

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(ii) If the spinner is spun twice, find the probability that the product of the two numbers is an odd number. **1**

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(iii) If the spinner is spun a total of eight times, find the probability that the marker lands on the number 6 five times. **1**

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Question 14 continues on the next page.

(iv) What is the fewest number of spins required to have at least a 99% chance of landing on 6 at least once? **2**

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The spinner is used by Don and Jon to play a simple game where for each spin a certain amount of winnings is award to one of the players as summarised in the following table:

X	4	5	6	7
Winnings	Don wins \$3	Jon wins \$6	Don wins \$4	Don wins \$2

(v) For each spin, which player is expected to win, and how much is the amount? **2**

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(vi) At what amount must the winnings for Jon be set such that the game presents a fair event, where the expected amount of winnings for both players is the same? **1**

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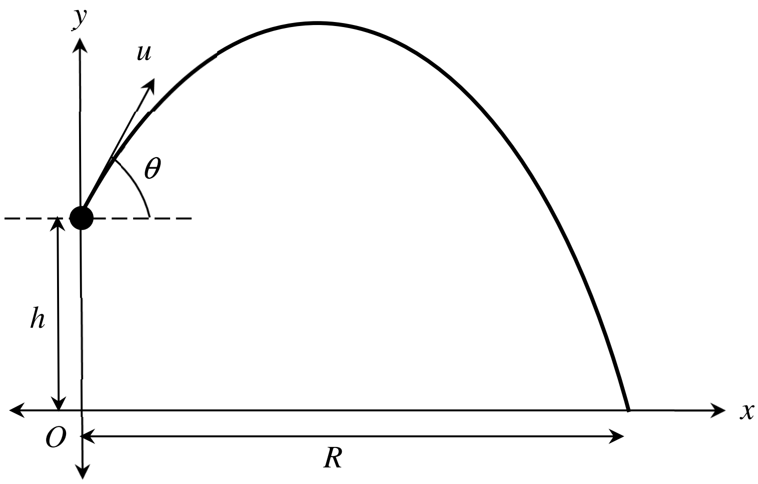
Question 14 continues on the next page.

(b) An object was projected into the air from a point h metres above O with initial velocity u m/s at an angle of θ from the horizontal, as shown in the diagram below.



Assuming gravity is g m/s^2, after t seconds the equation of the object's motion is given by:

$$y = -\frac{gx^2}{2u^2}\left(1 + \tan^2\theta\right) + x\tan\theta + h$$ (**DO NOT PROVE THIS.**)

(i) Show that: $R^2 = \left(\frac{u^4}{g^2} + 2h\frac{u^2}{g}\right) - \left(R\tan\theta - \frac{u^2}{g}\right)^2.$ **3**

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Question 14 continues on the next page.

(ii) Hence, or otherwise, show that the maximum horizontal range R_M is: **1**

$$\frac{1}{g}\sqrt{u^4 + 2ghu^2}\ .$$

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(iii) Show that the angle θ satisfies $\tan\theta = \dfrac{u^2}{gR_M}$. **1**

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(iv) Show that $\tan 2\theta = \dfrac{R_M}{h}$. **2**

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