Student details

Name: ______________________

Mark: ______________________

2025

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 6 – 14

90 marks

- Attempt Questions 11 – 34
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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1 What is the solution to $|x-2| > 5$?

(A) $x \in (-7,7)$

(B) $x \in [-3,7]$

(C) $x \in (-\infty,-7) \cup (7,\infty)$

(D) $x \in (-\infty,-3) \cup (7,\infty)$

2 Which of the following is a one-to-many relation?

(A) $y = x^3$

(B) $(x+2)^2 + (y-7)^2 = 121$

(C) $y^2 = x + 1$

(D) $y = x(x+3)^2$

3 A circle had an arc length of 4cm produced from an angle of 15° at the centre.

What is the circle's radius?

(A) $\frac{48}{\pi}$

(B) $\frac{4}{15}$

(C) $\frac{\pi}{3}$

(D) 6π

4 Which of the following represents the solutions to the equation: $4^x - 18\left(2^x\right) + 32 = 0$

(A) $x = 1, x = 4$

(B) $x = \log_2 4$, $x = \log_2 8$

(C) $x = 0, x = 3$

(D) $x = 0, x = 2$

5 The probability distribution of a discrete random variable X is summarised in the following table:

$\boldsymbol{x}$	–7	2	7	9	13
$\boldsymbol{P(x)}$	0.3	0.15	0.25	0.1	0.2

What is the variance of X?

(A) 3.45

(B) 7.586

(C) 21.446

(D) 57.5475

6 Two events, A and B, are independent where $P(A) = 0.32$ and $P(B) = 0.57$.

What is the value of $P\left(\overline{A} \cap \overline{B}\right)$?

(A) 0.1376

(B) 0.1824

(C) 0.2924

(D) 0.3876

7 Which of the following features of the function $y = f(x)$ will result in the trapezoidal rule underestimating the value of the integral $\int_a^b f(x)\,dx$ over the interval $[a,b]$?

(A) $f(x)$ is decreasing

(B) $f(x)$ is concave down

(C) $f(x)$ is concave up

(D) $f(x)$ is increasing

8 What is the value of the following geometric series, where $0 < \theta < \frac{\pi}{2}$:

$$\cos\theta + \cos\theta\sin^2\theta + \cos\theta\sin^4\theta + \ldots ?$$

(A) $\cos\theta\sin\theta$

(B) $\sec\theta$

(C) $\sin^2\theta$

(D) $\tan^2\theta$

9 Which of the following is equivalent to $\int 2^{5x}\,dx$?

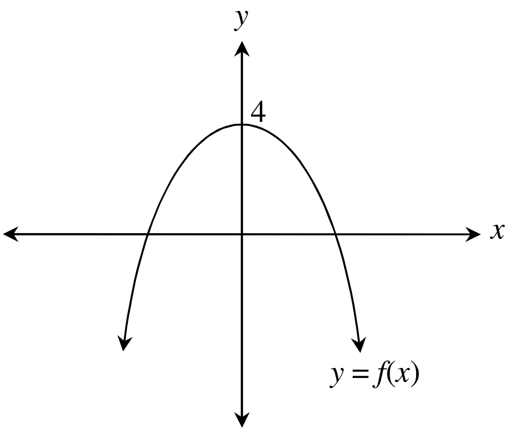
(A) $\dfrac{2^{5x}}{5\log_e 2} + C$

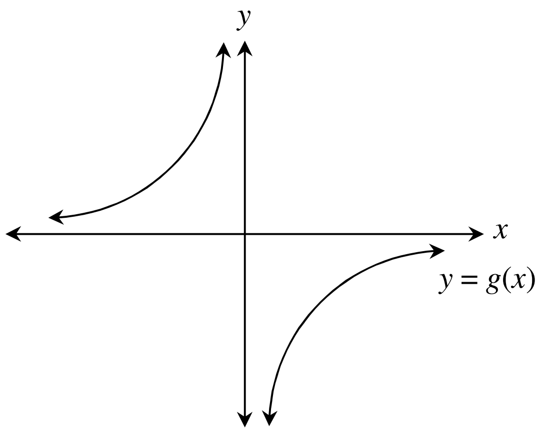
(B) $\dfrac{10^{5x}}{\log_e 2} + C$

(C) $\dfrac{5\left(2^{5x}\right)}{\log_e 2} + C$

(D) $\dfrac{5x\left(2^{5x}\right)}{\log_e 2} + C$

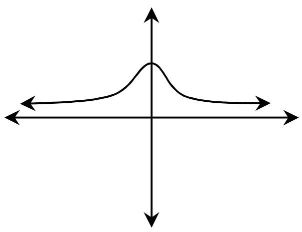
10 Consider the following functions:



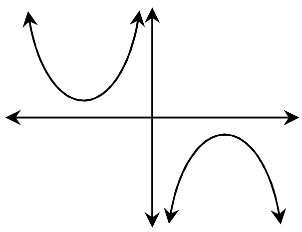


Which of the following graphs best represents the function $y = f\left[g\left(x\right)\right]$?

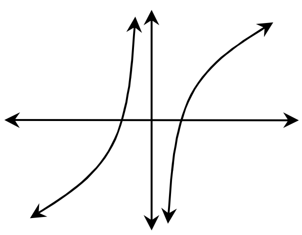
(A)



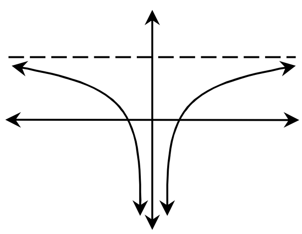
(B)



(C)



(D)



Section II

90 marks
Attempt Questions 11–34

In Questions 11–34, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Solve for x: $3x^2 - x - 2 = 0$ **1**

Question 12

Differentiate the following with respect to x:

(a) $y = 4\sqrt{x}$. **1**

(b) $y = \left(6x^2 - 5\right)^{13}$. **1**

(c) $y = e^{4x}\cos 2x$. **2**

(d) $y = \log_e\left(\dfrac{x+3}{x-5}\right)$. **2**

Question 13

Simplify the expression: $3\log_a x + \log_a x^9 - 4\log_a \sqrt{x}$. **2**

Question 14

Solve for x, $x \in [0, 2\pi]$: $3\tan x + \sqrt{3} = 0$. **2**

Question 15

The cost of using the public trains to a commuter depends on the number of stations travelled. A survey of 1000 commuters is summarised in the following table, where X is the discrete random variable representing the number of stops each commuter travelled and $P(x)$ represents the probability that one of the 1000 commuters had travelled x stations.

$X = x$	3	6	10	15	18
$P(x)$	0.18	0.3	0.43	0.05	a

(a) Find the value of a. **1**

(b) Estimate the number of commuters travelling less than or equal to 10 stations. **1**

(c) The cost associated with travelling on the public trains is given in the following table: **2**

$X = x$	3	6	10	15	18
Cost ($)	4.30	7.70	11.45	15.20	16.95

If a commuter was chosen at random, what is the expected cost of their travel? Round your solution to the nearest cent.

Question 16

Find:

(a) $\displaystyle\int \frac{5x^7}{3x^8 + 8}\,dx$. **1**

(b) $\displaystyle\int \sqrt{x}\left(4 - \sqrt{x}\right)dx$. **2**

(c) $\displaystyle\int_0^{\frac{\pi}{4}} \sin 2x + \sec^2 x \, dx$ **2**

Question 17

Find the number of terms in the series $180 + 169 + 158 + \ldots - 73$. **2**

Question 18

Find the domain and range of the function $f(x) = 4 - \sqrt{x-3}$, expressing your solution in set notation. **2**

Question 19

Differentiate $f(x) = \frac{4}{x}$ with respect to x, using first principles. **2**

Question 20

An experiment was conducted to observe the growth rate of bacteria under certain conditions. In this experiment, the bacteria population $P(t)$ over time t days followed the equation:

$$P(t) = P(0)e^{kt},$$

where the initial bacteria population $P(0)$ was 250 and k is a constant.

(a) If the bacteria population triples after 9 days, find the value of k, rounding your solution to three significant figures. **2**

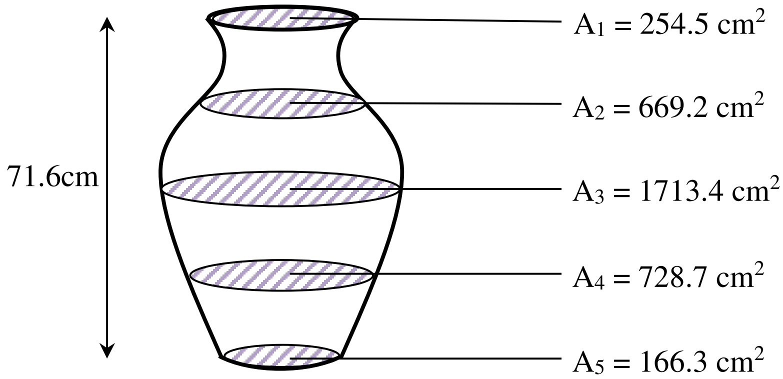
(b) Using your solution in (a), find the time it would take (to the nearest day) for the bacteria population to exceed 1500. **2**

Question 21

Prove the identify: $\dfrac{\tan\theta\sqrt{\dfrac{1-\cos^2\theta}{1+\tan^2\theta}}}{1-\sin^2\theta} = \tan^2\theta$. **3**

Question 22

A vase with height of 71.6cm had cross-sectional areas as shown in the diagram below. **2**



Using the trapezoidal rule, estimate the volume of the vase.

Question 23

The Maths and English results for a class of twelve students are summarised in the following table:

Maths (M)	81	76	56	98	57	62	85	76	90	44	59	68
English (E)	16	19	18	12	15	18	11	10	14	20	15	15

By using the table above, and rounding your solution to two decimal places where required:

(a) Determine the value of Pearson's correlation coefficient (r). **1**

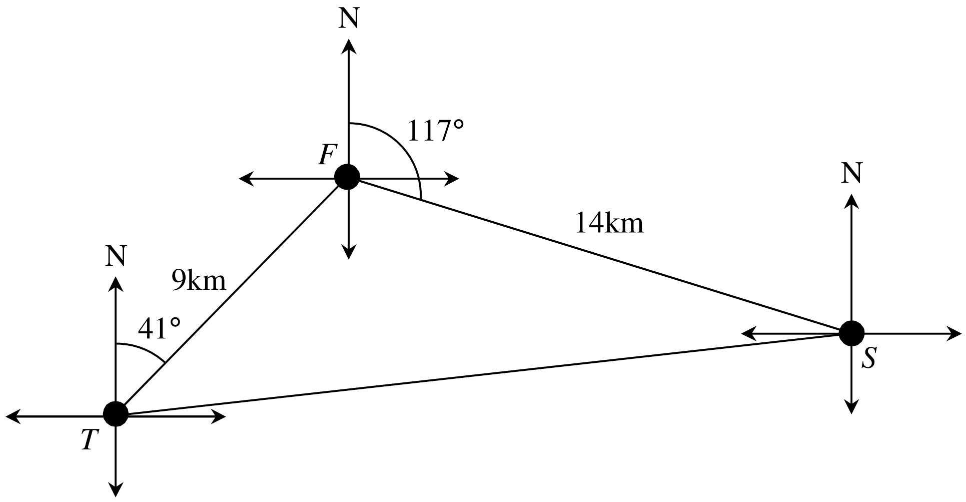
(b) Using (a), describe the strength of the correlation between E and M. **1**

(c) Find the 'line of best fit', stating your solution as $E = \boxed{} + \boxed{} \times M$. **2**

(d) Using your solution to part (c), estimate the Maths mark (nearest whole number) of a student who received an English mark of 17. **1**

Question 24

Sibby went for a morning hike, parking his car at Tilly's Bar (T) and walked 9 km on a bearing of 041°T to Fall's Lookout (F). After a short rest, he then proceeded to walk 14 km on a bearing of 117°T to The Two Sisters (S).



(a) Find the size of ∠TFS. **1**

(b) Find the distance Sibby walked to return to Tilly's Bar (T) from The Two Sisters (S), assuming he walked in a straight line. Round your solution to two decimal places. **2**

(c) Find the bearing of S from T, rounding your solution to the nearest degree. **2**

Question 25

A particle moves along a straight line, starting from the origin. The particle's displacement x (in metres) after time t seconds is given by the formula:

$$x = 2t - 3\log_e(t + 1).$$

(a) In terms of t, find an expression for the particle's velocity v. **1**

(b) Find the particle's initial velocity. **1**

(c) Find the time when the particle is at rest. **2**

(d) Find the distance travelled by the particle in the first three seconds, rounding your solution to four decimal places. **2**

Question 26

The high and low tides observed in *Whale Harbour* can be modelled using the formula:

$$h = 11 - 3\cos\left(\frac{\pi t}{6}\right),$$

where h represents the height of the tides in metres at time t hours. On a particular day in winter, the start of the model (when $t = 0$) occurs at 5am.

(a) Find the height of the water at high tide. **1**

(b) Find the period of motion assumed in the formula. **1**

(c) For safety reasons boats in Whale Harbour are only allowed to moor (park) when the height of the tide is above 9.5 metres. Find the times during the day when the boats are allowed to moor, rounding your solution to the nearest minute. **3**

Question 27

Consider the series $1 + \left(\sqrt{k} - 2\right) + \left(\sqrt{k} - 2\right)^2 + \left(\sqrt{k} - 2\right)^3 + \ldots$

(a) Explain why the largest integer value of k for the series to have a limiting sum is given by $k = 8$. **1**

(b) Find the limiting sum of the series when $k = 8$, leaving your solution with a rational denominator. **2**

Question 28

Consider the curve $y = x^2\sqrt{6 - x}$.

(a) Find where the curve crosses the x and y-axis. **2**

(b) Find any stationary point(s) and determine their nature. **3**

(c) Hence, or otherwise, sketch the curve $y = x^2\sqrt{6 - x}$ on a number plane, showing all key features. **2**

Question 29

Mario is a cultivator of exotic fungi (mushrooms) and has recently purchased an extensive piece of land with a large paddock dedicated to growing a special type of green mushroom known to give additional health to one who consumes it. Mario started his crop by planting 25,000 green mushrooms in the paddock. After frequent monitoring, it was found that the number of green mushrooms grew at a rate of 6.8% per month.

(a) Show that the number of green mushrooms in Mario's paddock after 3 years is 267,000 (rounded to the nearest thousand). **1**

After 3 years, Mario begins the harvesting process where he aims to harvest 19000 green mushrooms at the end of each month.

Let n be the number of months when harvesting begins and P_n be the number of green mushrooms in the paddock at the end of n months.

Assuming that the green mushrooms continue to grow at 6.8% per month,

(b) Using (a), show that: **2**

$$P_n = 267000 \times 1.068^n - \frac{19000\left(1.068^n - 1\right)}{0.068}.$$

(c) Find how many months from when harvesting began that the paddock runs out of green mushrooms, rounding your solution to the nearest month. **3**

Question 30

The probability density function $f(x)$ for a continuous random variable X is given by the following function

$$f(x) = \begin{cases} \dfrac{x}{6} & 1 \le x \le 3 \\ \dfrac{1}{3} & 3 < x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the mode for X. **1**

(b) Find the median for X, rounding your solution to three decimal places. **2**

(c) Find the function $F(x)$ that represents the cumulative frequency distribution. **3**

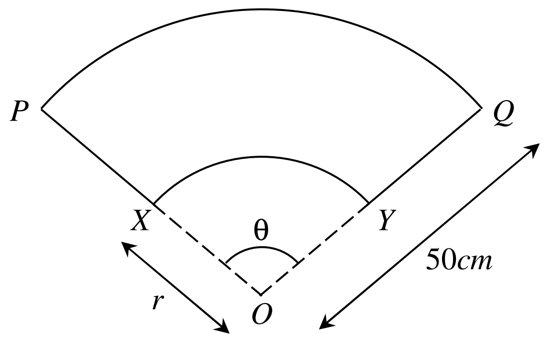
Question 31

$f(x)$ and $g(x)$ are two functions where $f\left(g(x)\right) = \dfrac{18x + 1}{23x + 1}$. **3**

If $f(x) = \dfrac{3x + 4}{4x + 5}$, find $g(x)$.

Question 32

Two arcs of a circle, *PQ* and *XY*, share a common centre O where the radius of the smaller arc *XY* is *r* cm and the radius of the larger arc *PQ* is 50 cm. Let $\angle POQ$ by θ (where θ is in radians), as shown in the diagram below.



The perimeter of *PQYX* is given to be 60 cm.

(a) Show that: **2**

$$\theta = \frac{2r - 40}{r + 50}.$$

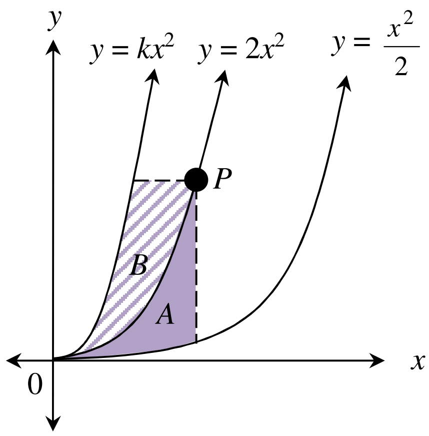
(b) Show that the area of *PQYX* is given by: **2**

$$A = 70r - 1000 - r^2.$$

(c) Find the value of θ that maximise the area of *PQYX*. **2**

Question 33

The following diagram shows the graphs $y = \dfrac{x^2}{2}$, $y = 2x^2$ and $y = kx^2$, where k is a constant. The two areas shown, A and B, are such that area A is double the area of B. Point P lies on the graph $y = 2x^2$. **3**



Find the value of k, rounding your solution to three decimal places.

End of paper