NEXT EDUCATION

2018

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 12

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

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1 What is the solution to x: $|x-2| = 2x-1$?

(A) $x = -3$

(B) $x = -1$

(C) $x = 1$

(D) $x = 3$

2 What is the size of the acute angle between the lines $y = 2x - 3$ and $3x + 5y - 1 = 0$?

(A) 32°

(B) 50°

(C) 82°

(D) 86°

3 What is the value of $2\int_{-2}^{3} |x-1|\, dx$?

(A) 11

(B) 5

(C) 13

(D) 17

4 What are the coordinates of the point that divides the interval AB into the ratio 4 : 3, given that the coordinates of A and B are (7,1) and (0,–6) respectively?

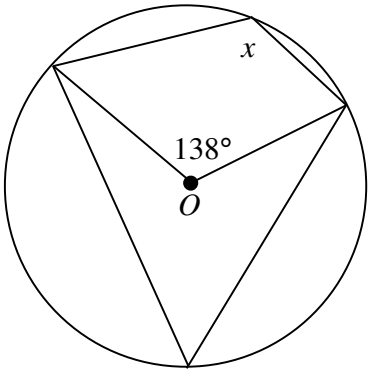
(A) (3,–3)

(B) (3,–2)

(C) (4,–2)

(D) (4,–3)

5 In the following diagram, O is the centre of the circle.



What is the value of x?

(A) 111°

(B) 42°

(C) 152°

(D) 69°

6 If $(x - 1)$ is a factor of the polynomial $P(x) = 2x^3 + x^2 + 2x + a$, what is the value of a?

(A) -6

(B) -5

(C) 5

(D) 6

7 How many unique arrangements of the letters in the word "CARRIAGE" if the two A's are placed next to each other?

(A) $8!$

(B) $\frac{8!}{2!}$

(C) $\frac{7!}{2!}$

(D) $\frac{6!}{2!}$

8 What is the volume of the solid formed when the area enclosed by the curve $y = \sec x$, the x-axis, and the lines $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x-axis?

(A) 4π units^2

(B) 2π units^2

(C) π units^2

(D) $\frac{\pi}{2}$ units^2

9 For a function $y = f(x)$, what is the derivative of $y = \log_e\left(\left[f(x)\right]^2\right)$?

(A) $\dfrac{2f'(x)}{\left[f(x)\right]^2}$

(B) $\dfrac{1}{\log_e\left(\left[f(x)\right]^2\right)}$

(C) $\dfrac{f'(x)}{2f(x)}$

(D) $\dfrac{2f'(x)}{f(x)}$

10 In terms of its displacement (x), a particle's acceleration (a) is given by the formula:

$$a = 4x - 4.$$

Initially, the particle is 2 metres to the right of the origin with a velocity (v) of 2 ms^{-1}

Which of the following is an expression for its velocity in terms of its displacement?

(A) $v = 4(x - 1)$

(B) $v = e^{2x}$

(C) $v = 2(x - 1)$

(D) $v = e^{4x}$

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question on a NEW page on your OWN PAPER.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW page on your OWN PAPER.

(a) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x}{4x}$. **2**

(b) Differentiate $y = \sin^{-1}\left(x^3\right)$. **2**

(c) Solve for x: $\dfrac{4x-3}{2x+1} \geq 1$. **3**

(d) Use the substitution $u = 4 - x^3$ to evaluate $\int_{-1}^{1} x^2\sqrt{4-x^3}\ dx$. **3**

(e) Prove the following identity: $\dfrac{\sin x}{1-\cos x} = \cot\dfrac{x}{2}$. **2**

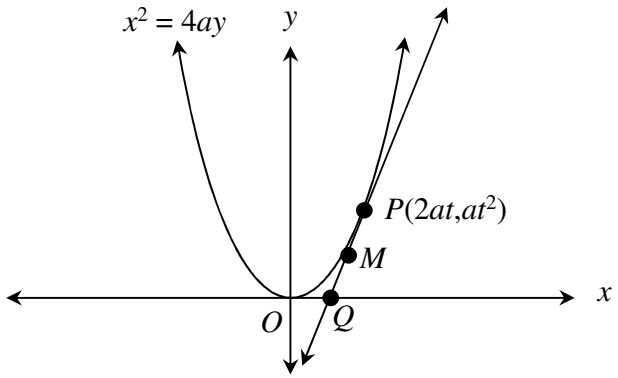
(f) Find the term independent of x in the expansion of $\left(\dfrac{3x^2}{2} - \dfrac{1}{3x}\right)^9$. **3**

End of Question 11.

Question 12 (15 marks) Use a NEW page on your OWN PAPER.

(a) Find the exact value of $\cos\left(2\tan^{-1}\frac{1}{4}\right)$. **3**

(b) The point $P(2at, at^2)$ lies on the parabola $x^2 = 4ay$. The tangent at P cuts the x-axis at the point Q, where the midpoint of PQ is M.



(i) Show that the equation of the tangent at P is $y = tx - at^2$. **1**

(ii) Find the coordinates of Q. **1**

(iii) Find the equation of the locus of M as P moves on the parabola. **2**

(c) (i) Sketch the function: $y = 2\tan^{-1}x$. **1**

(ii) For the function: $y = 2\tan^{-1}x - \frac{x}{4}$

(α) Show graphically why the function only has one root for $x > 0$. **1**

(β) Taking $x = 10$ as a first approximation to this root, use one application of Newton's method to find a better approximation, rounding your solution to two decimal places. **2**

Question 12 continues on the next page.

(d) A group of ten friends arrive at a café for lunch where the only available seating is two circular tables, one that seats six people and another that seats four.

If all ten friends were to be seated in the two available tables,

(i) How many seating arrangements are there? **2**

(ii) What is the probability that a particular couple are to be seated next to each other? **2**

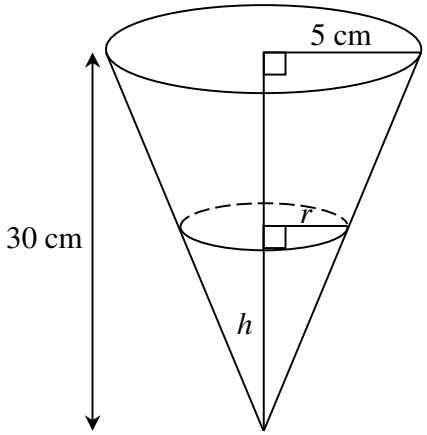
End of Question 12.

Question 13 (15 marks) Use a NEW page on your OWN PAPER.

(a) Use mathematical induction to prove for all integers $n \geq 1$: **3**

$$\frac{1}{3\times 4\times 5}+\frac{2}{4\times 5\times 6}+\ldots+\frac{n}{(n+2)(n+3)(n+4)}=\frac{1}{6}-\frac{1}{n+3}+\frac{2}{(n+3)(n+4)}.$$

(b) An inverted conical vase had dimensions comprising a base of 5cm and height of 30cm. Water is poured into the vase at a rate of $2\text{cm}^3/\text{min}$, where r is the radius and h is the height of the water at any point, as shown in the diagram below.



(i) Show that $r=\frac{h}{6}$. **1**

(ii) Find the rate at which the level of water is rising when the water is 10cm deep, rounding your answer to one decimal place. **3**

(c) A particle moves along a straight line such that its displacement x metres from a fixed point O after t seconds is given by:

$$x=2\cos 3t-\sqrt{12}\sin 3t.$$

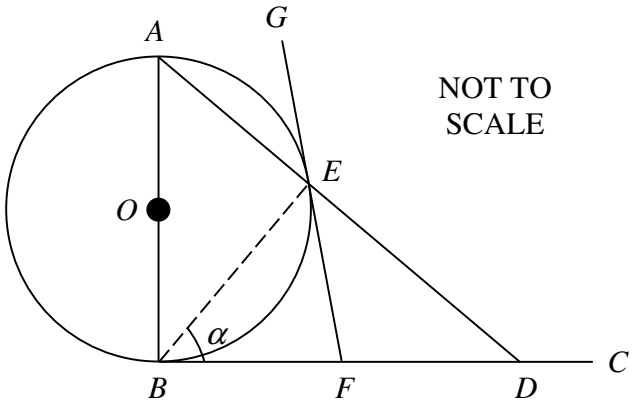
(i) Show that the particle moves with simple harmonic motion. **2**

(ii) If $x=R\cos(3t+\alpha)$, find the exact values for R and α. **1**

(iii) Find the particle's maximum velocity. **2**

Question 13 continues on the next page.

(d) In the following diagram, AB is a diameter of the circle with centre O and BC is a tangent to the circle at B. The line AED intersects the circle at E and BC at D. The tangent to the circle at E intersects BC at F. Let $\angle EBF = \alpha$. **3**



Copy the diagram into your writing booklet.

Prove that $\angle FED = 90° - \alpha$.

End of Question 13.

Question 14 (15 marks) Use a NEW page on your OWN PAPER.

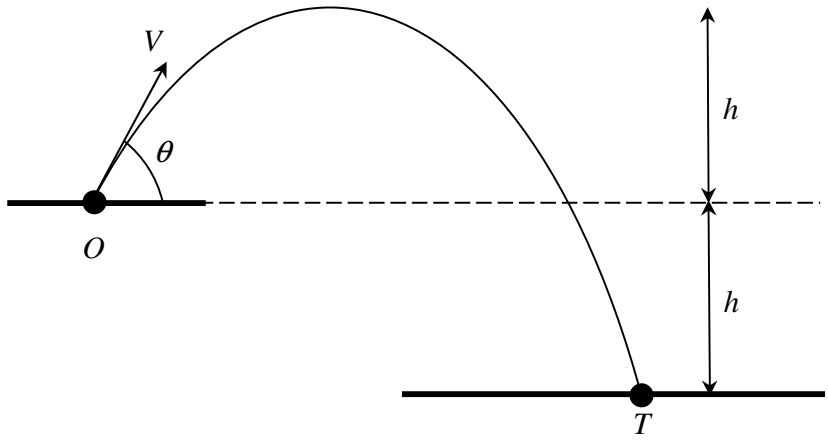
(a) Differentiate: $y = 6^x$ **2**

(b) By considering the expansion of $(1 + x)^{2n}$, show that: **3**

$$^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_{n-1} = 2^{n-1} - \frac{(2n)!}{2(n!)^2}$$

where n is a positive integer.

(c) An object was projected into the air from O with initial velocity V m/s at an angle of θ, reaching a maximum vertical height of h metres. The object strikes its intended target T located h metres below O, as shown in the diagram below.



After t seconds, the horizontal (x) and vertical (y) displacements of the ball is given as follows (**DO NOT PROVE THESE**):

$$x = V t\cos\theta \qquad \text{and} \qquad y = -\frac{gt^2}{2} + V t\sin\theta$$

where gravity is g m/s^2.

Question 14 continues on the next page.

(i) Prove that $h = \dfrac{V^2 \sin^2 \theta}{2g}$. **2**

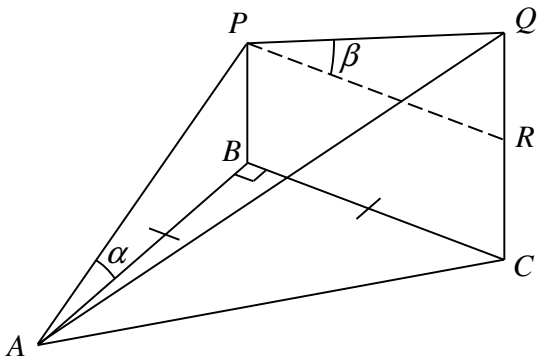
(ii) Show that the time taken for the object to reach the target T is: **2**

$$\frac{V \sin \theta \left(1+\sqrt{2}\right)}{g} \text{ seconds.}$$

(iii) Hence, show that the horizontal distance travelled from O to T is: **1**

$$\frac{V^2 \sin 2\theta \left(1+\sqrt{2}\right)}{2g} \text{ metres.}$$

(d) ABC is a right-angled, isosceles triangle where $AB = BC$ and $\angle ABC = 90°$. P is vertically above B and Q is vertically above C. The angle of elevation of P from A is α and the angle of elevation of Q from P is β.



(i) If the angle of elevation of Q from A is θ, show that: **2**

$$\tan\theta = \frac{\tan\alpha + \tan\beta}{\sqrt{2}}.$$

(ii) If $\angle APQ = \phi$, show that: **3**

$$\cos\phi = -\sin\alpha\sin\beta.$$

End of paper.