

2016

TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used

Total marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 16

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I**10 marks****Attempt Questions 1 – 10****Allow about 15 minutes for this section**Use the multiple choice answer sheet for Questions 1 – 10

1 What is $\frac{3e + \pi^2}{6(\sqrt{9.5} + 8.5^2)}$ to 4 significant figures?

- (A) 0.04
- (B) 0.0399
- (C) 0.03988
- (D) 0.039878

2 Solve for x : $x + \frac{4}{x} = 3$.

- (A) $x = 1, 3$
- (B) $x = 1, 4$
- (C) $x = 3, 4$
- (D) No real solution for x

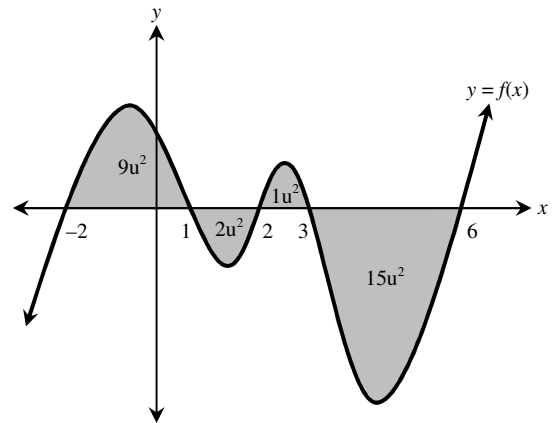
- 3 What is the value of $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$?
- (A) 3
(B) 6
(C) 9
(D) ∞
- 4 Which of the following DOES NOT EQUAL to $\sqrt{4^{8x}}$?
- (A) 2^{8x}
(B) 4^{4x}
(C) 8^{2x}
(D) 256^x
- 5 What is the value of θ , where $0 \leq \theta \leq \pi$ $-2\cos\theta = \sqrt{3}$?
- (A) $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$
(B) $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$
(C) $\theta = \frac{5\pi}{6}$ only
(D) $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

- 6 The following diagram shows the graph $y = f(x)$, with the area of certain sections labelled.

Using the graph, evaluate:

$$\int_{-2}^3 f(x) dx + \int_1^6 f(x) dx$$

- (A) -7
- (B) -8
- (C) 7
- (D) 8



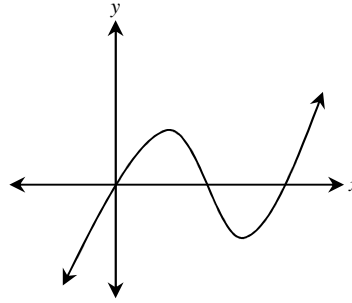
- 7 What is the coordinates of the vertex in the parabola $y - 2 = x^2 - 6x$?

- (A) (6, -2)
- (B) (-6, 2)
- (C) (3, -7)
- (D) (-3, 7)

- 8 If $\int_1^7 g(x) dx = 9$, what is the value of $\int_1^7 5 - 2g(x) dx$?

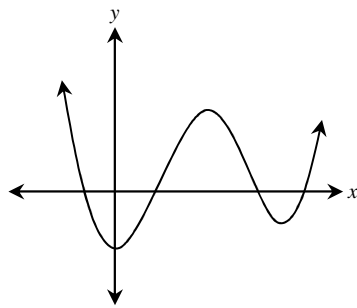
- (A) 8
- (B) 10
- (C) 12
- (D) There is not enough information to derive a value.

9 The following depicts the graph of the function $y = f(x)$:

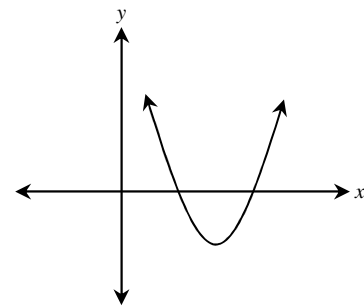


Which of the following graphs best represents $y = f''(x)$?

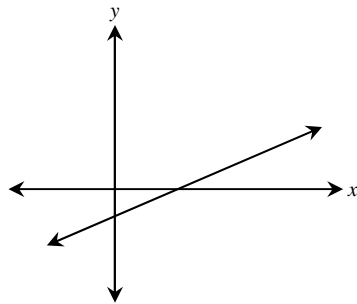
(A)



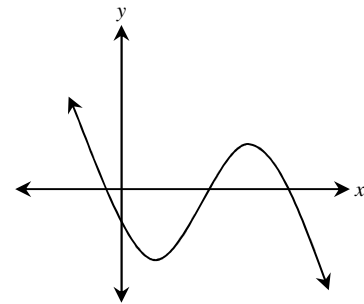
(B)



(C)



(D)



10 A particle is moving along the x -axis in a straight line. After t seconds, the particle's velocity and acceleration is -12 ms^{-1} and 2 ms^{-2} respectively.

Which statement best describes the motion of the particle after t seconds?

- (A) The particle is moving to the left and is slowing down.
- (B) The particle is moving to the right and is slowing down.
- (C) The particle is moving to the left and is speeding up.
- (D) The particle is moving to the right and is speeding up.

Section II**90 marks****Attempt Questions 11 – 16****Allow about 2 hours and 45 minutes for this section**

Answer each question on a NEW page on your OWN PAPER.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW page on your OWN PAPER.

- (a) State the exact value of $\tan \frac{5\pi}{3}$. **1**
- (b) Express $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ with a rational denominator. **2**
- (c) Differentiate the following with respect to x :
- (i) $e^{\sin x}$. **1**
- (ii) $\frac{\log_e(2x)}{x^2+4}$. **2**
- (d) Factorise $2x^2 + x - 3$. **1**
- (e) Solve for x : $\frac{3x+4}{6} - \frac{2x-1}{5} = 2$. **2**

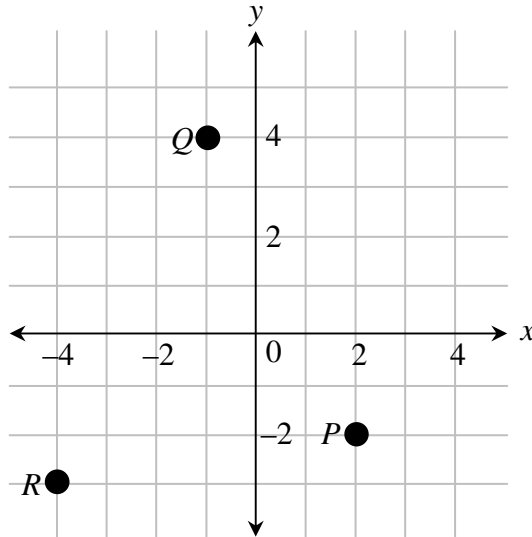
(f) Solve for x : $|2x + 7| = |6x - 4|$. **2**

(g) Sketch the curve of $y = \frac{\sin 2\pi}{4}$ for $0 \leq x \leq 2\pi$, labelling all key features. **2**

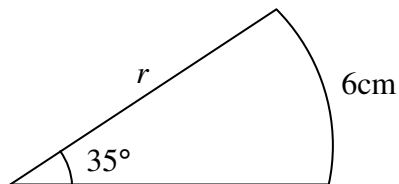
(h) If $\sin \theta = \frac{3}{8}$ and $\cos \theta < 0$, find the exact value of $\tan \theta$. **2**

Question 12 (15 marks) Use a NEW page on your OWN PAPER.

- (a) Points $P(2,-2)$, $Q(-1,4)$ and $R(-4,-3)$ lie on a Cartesian plane, as shown in the diagram below.



- (i) Find the length of the interval PQ . 1
 - (ii) Find the equation of a straight line passing through P and Q , leaving your answer in general form. 2
 - (iii) Find the perpendicular distance between the point R and the line passing through P and Q . 2
 - (iv) Hence, or otherwise, find the area of $\triangle PQR$, leaving your answer in exact form. 1
- (b) The following diagram shows a sector of a circle. 2



Find the length of the radius r as shown in the diagram, correct to 3 decimal places.

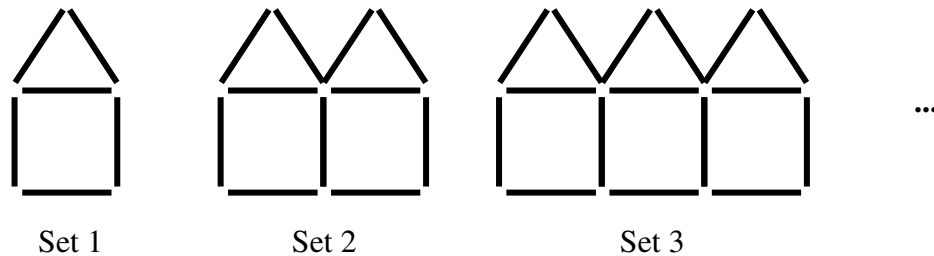
- (c) Evaluate $\int_0^{\sqrt{2}} \frac{4}{\sqrt{4-x^2}} dx$. **2**
- (d) Find the equation of the tangent to the curve $y = x \sin x$ at the point where $x = \frac{\pi}{4}$. **2**
- (e) Solve for x : $9^{x+1} - 28(3^x) + 3 = 0$. **3**

Question 13 (15 marks) Use a NEW page on your OWN PAPER.

(a) Find $\int \frac{1+x}{1+x^2} dx$. **2**

(b) Estimate to 1 decimal place $\int_2^4 \ln x dx$ using Simpson's rule with 5 function values. **2**

(c) Small identical sticks are used to make the following sets of patterns:

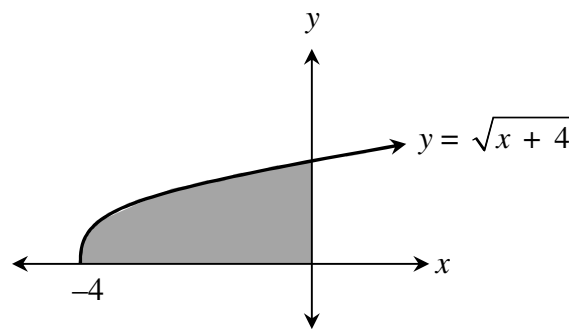


(i) How many sticks are required to make set 15? **1**

(ii) How many sticks are required to make *all sets* between 1 and 15 (inclusive)? **2**

(iii) What is the highest number of sets that can be made with 7000 sticks (starting from Set 1)? **2**

(d) The following diagram shows the graph $y = \sqrt{x+4}$: **2**



Find the area of the shaded region.

- (e) Having recently started university, Ko's parents decide to help her pay for her annual school fees of \$5550 paid at the end of each year. To do this, they deposit a one-off amount of \$ P into a bank earning interest of 6% per annum (compounding annually). Ko's chosen course is a degree that takes 12 years to complete and the amount of \$ P is intended to exactly provide for the entire amount of school fees over this period.

Letting A_n be the amount remaining at the end of n years,

- (i) Show that $A_n = (P - 92500) \times 1.06^n + 92500$. **3**
- (ii) Find the amount \$ P correct to nearest dollar. **1**

Question 14 (15 marks) Use a NEW page on your OWN PAPER.

- (a) A particle moves along a straight line where its velocity $v \text{ ms}^{-1}$ after time t seconds is given by the formula:

$$v = 3t^2 - 42t + 135.$$

Initially, the particle is 12 metres to the right of the origin O.

- (i) In terms of t , find an expression for the particle's acceleration a in ms^{-2} . **1**
- (ii) In terms of t , find an expression for the particle's displacement x in metres. **1**
- (iii) Find when and where the particle is at rest. **2**
- (iv) Find when the particle reaches its minimum velocity and state its value. **2**
- (b) (i) Solve for x to 3 decimal places: $e^x = 8$. **1**
- (ii) Solve for x : $2\log x = \log(x + 6)$. **1**
- (iii) Simplify the following: $\log_5 3 + \log_{25} 81$. **2**
- (c) Po and Ra played a game with a die with 6 faces (1, 2, 3, 4, 5 and 6), where Po had the first roll. Po wins the game if the number was less than 3, otherwise Ra has his turn. If Ra rolls an odd number he wins, else Po has her turn and the game is repeated. The game is played until there is a winner.
- (i) What is the probability that Po wins on her *first* roll? **1**
- (ii) What is the probability that Po wins *overall*? **2**
- (iii) What is the probability that Po has at least 2 rolls before the game ends? **2**

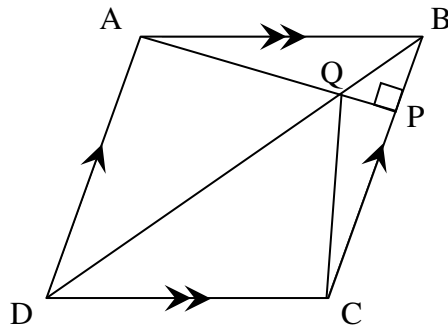
Question 15 (15 marks) Use a NEW page on your OWN PAPER.

- (a) The mass M (in g) of a radioactive substance decays over time t (in days) according to the formula:

$$M = M_0 e^{-kt}$$

where M_0 is the initial mass of the radioactive substance and k is a positive constant.

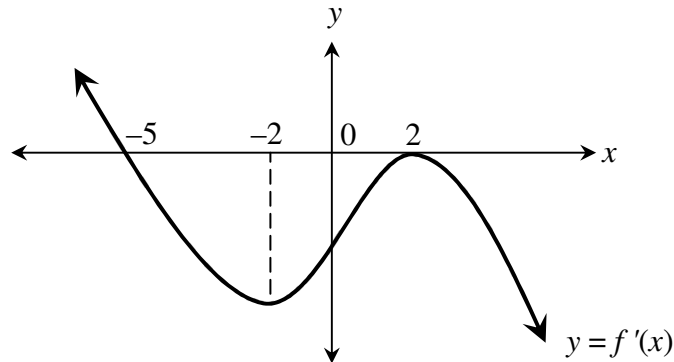
- (i) Show that M satisfies the differential equation $\frac{dM}{dt} = -kM$. 1
- (ii) A 10g of this radioactive substance loses 25% of its mass after 15 days.
- (α) Find the value of k rounding to two significant figures. 2
- (β) Using (α), find the 'half-life' of the radioactive substance. i.e. the time it takes for the substance to lose half its mass (nearest day). 2
- (b) In the diagram, $ABCD$ is a rhombus, AP is perpendicular to BC and intersects BD at Q .



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- (i) Explain why $\angle ADB = \angle CDB$. 1
- (ii) Prove that the triangles AQD and CQD are congruent. 2
- (iii) Show that $\angle DCQ$ is a right-angle. 2

(c) The following diagram shows the graph $y = f'(x)$:

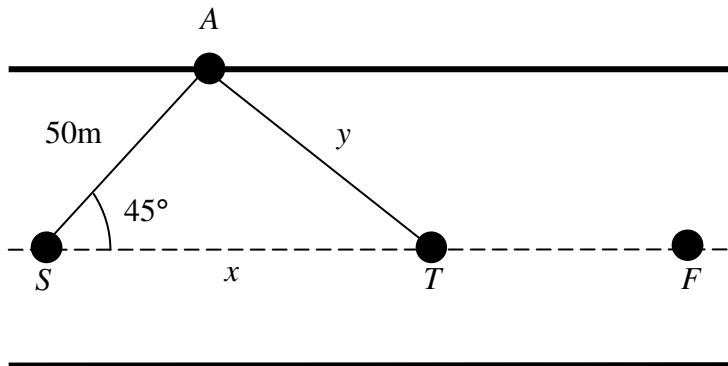


- (i) State the values of x where the graph $y = f(x)$ is increasing? **1**
- (ii) Draw a neat sketch of $y = f''(x)$, showing all key features. **2**
- (iii) Given that the y -intercept of the graph $y = f(x)$ is 7, draw a neat sketch of $y = f(x)$, showing all key features. **2**

Question 16 (15 marks) Use a NEW page on your OWN PAPER.

(a) State the coordinates of the focus for the parabola: $y = -x^2 - 6x - 4$ **3**

(b) Apo is a spectator observing a rowing time trial event where competitors row from the starting point S to the finishing point F . Apo is seated on the river bank at point A 50 metres from point S in a north-easterly direction, i.e. $\angle ASF = 45^\circ$.



The last competitor of the day, with average pace of $4\sqrt{2}$ metres per second, was about to commence his race. T is a point during the race where the competitor has covered x metres, i.e. $ST = x$, and the distance between Apo and the competitor is y metres, i.e. $AT = y$.

(i) Show that: $y^2 = x^2 - 50\sqrt{2}x + 2500$. **2**

(ii) What is the distance between Apo and the competitor after 25 seconds? Express your answer in exact values. **1**

(iii) At what rate is the distance between Apo and the competitor increasing at after 25 seconds? Express your answer in exact values. **3**

- (c) Two men were hired by a company to deliver a special package to an address in a remote village. The return trip covers a total distance of 1600km. The men are each paid \$45 per hour for the trip by the company.

The plane used for this trip consumed fuel at a rate of $\left(25 + \frac{v^2}{10}\right)$ litres per hour, where v km per hour is the average speed for the whole trip. The cost of the fuel used by the plane is \$0.80 per litre, all of which is borne by the company.

- (i) Show that the cost to the company (\$ C) for the total trip is given by: **3**

$$C = \frac{176000}{v} + 128v.$$

- (ii) Find the minimum cost that the company can pay (nearest dollar), and state the average speed at which this cost is achieved (to one decimal place). **3**

End of paper.