NEXT EDUCATION

2019

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used

Total marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 15

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

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1 Which of the following are the solutions for x: $x^2 - x - 6 = 0$?

(A) $x = 6, 1$

(B) $x = 3, -2$

(C) $x = -3, 2$

(D) $x = -6, -1$

2 Which of the following is a solution for x in the equation: $\sqrt{3}\tan x + 1 = 0$

(A) $x = \frac{\pi}{6}$

(B) $x = \frac{2\pi}{3}$

(C) $x = \frac{5\pi}{3}$

(D) $x = \frac{11\pi}{6}$

3 Which of the following is equivalent to $\left(3\sqrt{5} - 2\sqrt{3}\right)^2$?

(A) 33

(B) 57

(C) $57 - 12\sqrt{15}$

(D) $33 - 6\sqrt{15}$

4 Which of the following is $\log_3 4$ rounded to three significant figures?

(A) 0.79

(B) 0.792

(C) 1.26

(D) 1.262

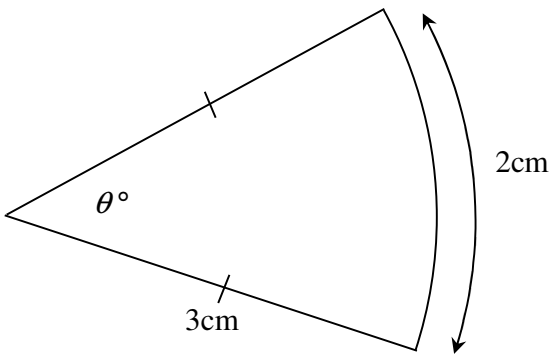
5 Which of the following is equal to the value of θ?

(A) $\theta = \frac{2}{3}$

(B) $\theta = \frac{3}{2}$

(C) $\theta = \frac{2 \times 180}{3\pi}$

(D) $\theta = \frac{3\pi}{2 \times 180}$



6 Given that $y = f(x)$ is an even function and $y = g(x)$ is an odd function, which of the following is equivalent to the integral: $\int_{-a}^{a} f(x) - g(x)\, dx$ where a is a constant.

(A) $2a$

(B) $2\int_{0}^{a} f(x)\, dx$

(C) $2\int_{0}^{a} g(x)\, dx$

(D) $\int_{0}^{a} f(x) + g(x)\, dx$

7 The first three terms of a series is 2, x, 18.

What could the value of x be if the series followed a geometric progression?

(A) –6

(B) –8

(C) 10

(D) 12

8 Which of the following is equal to $\int_{0}^{4} 2^{x}\, dx$ using Simpson's Rule with five function values?

(A) 11.67

(B) 17.67

(C) 21.67

(D) 26.5

9 Which of the following is the derivative of $\ln(\ln x)$?

(A) $\dfrac{1}{x}$

(B) $\dfrac{1}{\ln x}$

(C) $\dfrac{x}{\ln x}$

(D) $\dfrac{1}{x \ln x}$

10 Sarah was 20 years from retirement and decided to start set aside an additional \$500 at the start of each month. She found a superannuation company that offered her 6% p.a. compounding monthly. After month n, the superannuation company increased the interest charged to her fund to 9% p.a, applying to the \$500 deposited during month $(n + 1)$ and thereafter.

Which of the following is equivalent to the total amount of Sarah's fund at the point of her retirement?

(A) $500(1.005)^{n}(1.0075)^{240-n}$

(B) $500(1.005)\left(\dfrac{1.005^{n}-1}{0.005}\right) + 500(1.0075)\left(\dfrac{1.0075^{n}-1}{0.0075}\right)$

(C) $500(1.005)\left(\dfrac{1.005^{n}-1}{0.005}\right)1.0075^{240-n} + 500(1.0075)\left(\dfrac{1.0075^{240-n}-1}{0.0075}\right)$

(D) $500(1.005)^{n}\left(\dfrac{1.005^{n}-1}{0.005}\right) + 500(1.0075)^{240-n}\left(\dfrac{1.0075^{240-n}-1}{0.0075}\right)$

End of Section I.

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question on a NEW page on your OWN PAPER.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW page on your OWN PAPER.

(a) Factorise: $m^3 - 125$. **1**

(b) Solve for x: $\dfrac{x-8}{x+1} \leq 0$. **2**

(c) Find the limiting sum of the series: $20 + 5 + 1.25 + \ldots$ **2**

(d) Evaluate $\lim\limits_{x \to \infty} \dfrac{3x^5 - 8x^3 + 5}{5x^2 + 7x^5}$. **2**

(e) Differentiate the following with respect to x:

(i) $y = 9x^8$. **1**

(ii) $y = e^{\sin x}$. **1**

(iii) $y = x^3 e^{3x}$. **2**

Question 11 continues on the next page.

(f) Solve for x: $|x-1| = 2x+4$. **2**

(g) If $\sec\theta = -\frac{6}{5}$ and $\tan\theta < 0$, find the exact value of $\sin\theta$. **2**

End of Question 11.

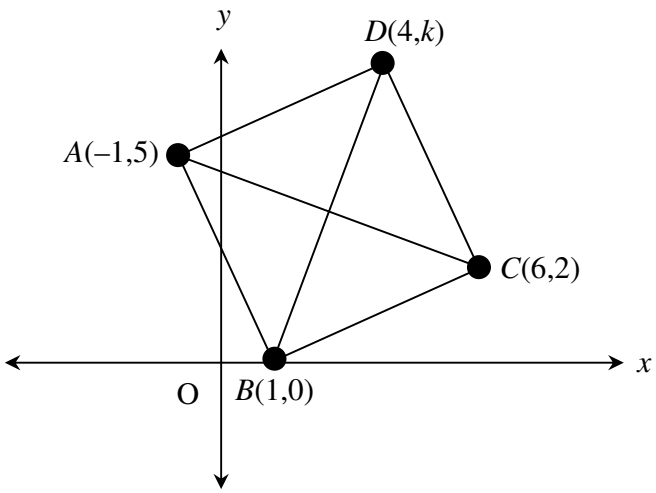
Question 12 (15 marks) Use a NEW page on your OWN PAPER.

(a) Find:

(i) $\int \frac{x}{x^2 + 3}\, dx$. **1**

(ii) $\int \frac{x^2 + 3}{x}\, dx$. **2**

(b) $ABCD$ is a square with vertices $A(-1,5)$, $B(1,0)$, $C(6,2)$ and $D(4,k)$, as shown in the diagram below.



(i) Find the gradient of AC. **1**

(ii) Show that the equation of BD is $7x - 3y - 7 = 0$. **2**

(iii) Find the value of k, the y-coordinate of D. **1**

(iv) Find the length of AC. **1**

(v) Hence, or otherwise, find the area of $ABCD$. **1**

Question 12 continues on the next page.

(c) Show that $\int_0^{\ln 3} e^{2x}\,dx = 4.$ **3**

(d) Solve for x: $2\left(x^2+1\right)^2 - 19\left(x^2+1\right) - 10 = 0.$ **3**

End of Question 12.

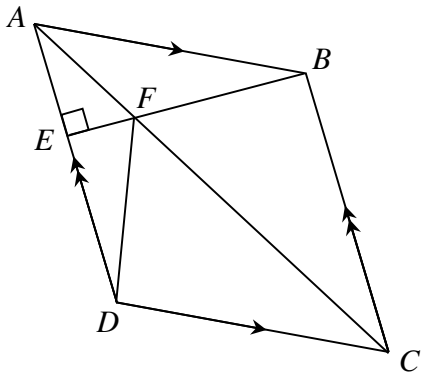
Question 13 (15 marks) Use a NEW page on your OWN PAPER.

(a) Consider the parabola: $y = x^2 - 4x$.

(i) Find the coordinates of the focus. **2**

(ii) Find the equation of the directrix. **1**

(b) $ABCD$ is a rhombus, where BE is perpendicular to AD and intersects AC at F. This is shown in the diagram below.



NOT TO SCALE

Copy the diagram into your answer booklet.

(i) Explain why $\angle BCA = \angle DCA$. **1**

(ii) Prove that $\Delta BFC \equiv \Delta DFC$. **3**

(iii) Show that $\angle FBC$ is a right angle. **1**

(iv) Hence, or otherwise, find the size of $\angle FDC$. **1**

(c) Prove the identity: $\dfrac{1+\cot x}{\operatorname{cosec} x} - \cos x = \dfrac{\sec x}{\tan x + \cot x}$. **3**

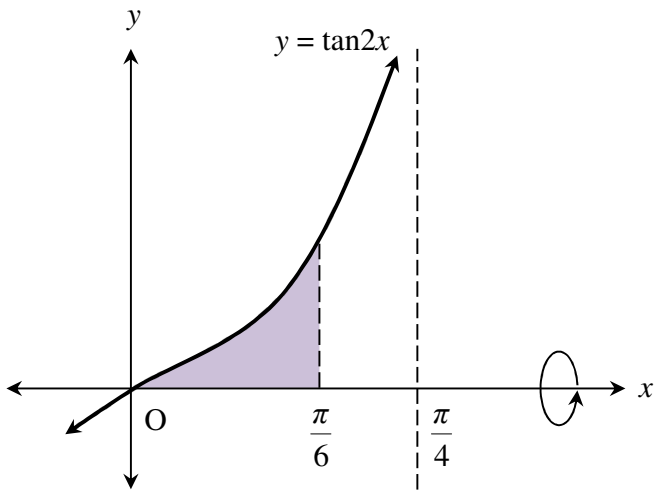
(d) Find the equation of the normal to the curve $y = 3e^{2x}$ at the point where $x = 1$. **3**

End of Question 13.

Question 14 (15 marks) Use a NEW page on your OWN PAPER.

(a) An unbiased die with faces showing 1, 2, 3, 4, 5 and 6 was rolled four times and the value of each roll was added together. What is the probability that the sum is greater than 5? **2**

(b) The following diagram shows the area enclosed between the curve $y = \tan 2x$, the x-axis and the line $x = \frac{\pi}{6}$.



The area shown is rotated about the x-axis to form a solid.

(i) Show that the volume V of the solid formed is given by: **2**

$$V = \pi \int_0^{\frac{\pi}{6}} \sec^2 2x - 1 \, dx .$$

(ii) Hence, find the volume of the solid formed. **2**

Question 14 continues on the next page.

(c) The velocity v in m/s of a particle moving along a straight line after time t seconds is given by the formula:

$$v = 4t^3 - 6t.$$

The particle is initially 4m to the left of the origin.

(i) In terms of t, find an expression for the particle's acceleration a in m/s^2. **1**

(ii) What is the particle's initial acceleration? **1**

(iii) In terms of t, find an expression for the particle's displacement x in metres. **2**

(iv) When is the particle at the origin? **2**

(d) Find the value of a and b: $\log_3 2 + \log_9 16 = \log_a b$. **3**

End of Question 14.

Question 15 (15 marks) Use a NEW page on your OWN PAPER.

(a) The equation $x^2 - 4x + 8 = 0$ has roots $x = \alpha$ and β. **2**

Find the value of $\alpha^3 + \beta^3$.

(b) The number of bacteria N grown in a laboratory for a particular experiment after t seconds is given by the equation:

$$N = 35000e^{0.004t}.$$

(i) What is the initial number of bacteria used in the experiment? **1**

(ii) Find the number of bacteria present after 12 seconds. Round your solution to the nearest whole number. **1**

(iii) How long would it take for the bacteria to triple in number? Round your solution to the nearest second. **2**

(iv) At what rate is the number of bacteria increasing at after 12 seconds? Round your solution to the nearest whole number. **2**

(c) Consider the curve $y = e^{-x} - e^{-2x}$.

(i) Find the y-intercept. **1**

(ii) Find the coordinates of the stationary points and determine their nature. Leave your solution in exact form. **3**

(iii) State the values of x where the curve is monotonically decreasing. **1**

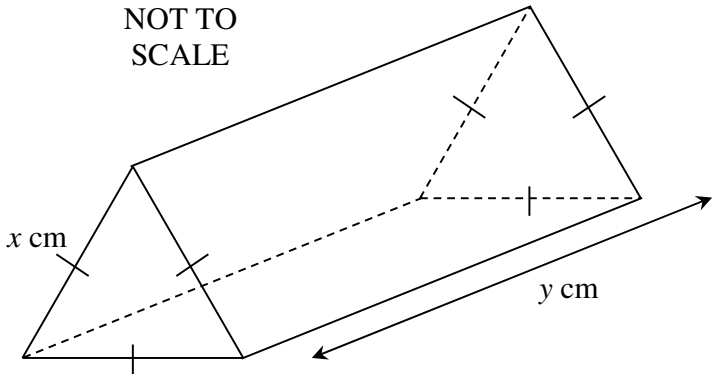
(iv) Explain the behaviour of the curve for large values of x. **1**

(v) Sketch the curve $y = e^{-x} - e^{-2x}$. **1**

End of Question 15.

Question 16 (15 marks) Use a NEW page on your OWN PAPER.

(a) A triangular prism has an equilateral triangle base with sides of x cm and a length of y cm, as shown in the diagram below.



The volume of the prism is 1000 cm^3.

(i) Show that $y = \dfrac{4000}{\sqrt{3}x^2}$. **2**

(ii) Hence, show that the surface area of the prism A cm^2 is given by: **2**

$$A = \frac{4000\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}.$$

(iii) Hence, find the value of x that minimises the surface area of the prism, rounding your solution to one decimal place. **3**

Question 16 continues on the next page.

(b) To renovate the castle of *Winterfall*, *Sansi* borrowed \$900,000 from the *Iron Bank* at an interest rate of 6% per annum, where interest is charged monthly and repayments of \$5400 are made at end of each month.

The amount owing on the loan after n months is denoted as A_n.

(i) Show that $A_3 = 900000 \times 1.005^3 - 5400 \times (1 + 1.005 + 1.005^2)$. **1**

(ii) Hence, show that $A_n = 1080000 - 180000 \times 1.005^n$. **2**

(iii) It can be shown that the amount owing on the loan after 12 years (A_{144}) is \$710,865. (**DO NOT PROVE THIS**). **3**

At the end of 12 years, the *Iron Bank* increases the interest rate to 7.2% per annum. How long, from the start of the loan, will it take *Sansi* to fully repay the loan if her repayments were to remain unchanged at \$5400?

(iv) Taking into account the increase in interest rate, what would be the level of monthly repayments required if *Sansi* wanted to fully repay the loan after 20 years? Round your solution to the nearest dollar. **2**

End of paper.