NExT EDUCATION

2017

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 12

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

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1 Which of the following is a simplified expression for $\dfrac{\sin 2x}{1-\cos 2x}$?

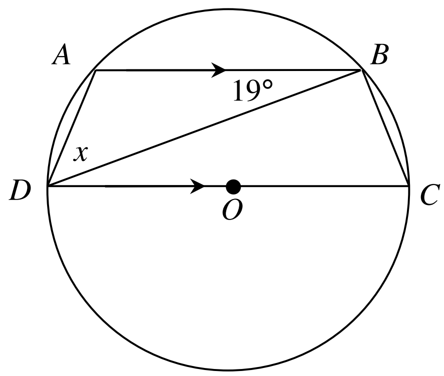
(A) $\sin x$

(B) $\cos x$

(C) $\tan x$

(D) $\cot x$

2 In the following diagram, O is the centre of the circle. What is the value of x?



(A) 19°

(B) 38°

(C) 52°

(D) 71°

3 What is the point that divides the interval AB into the ratio 3 : 2, given that the coordinates of A and B are (–1,1) and (4,11) respectively?

(A) $\left(\frac{6}{5},5\right)$

(B) (7,2)

(C) $\left(5,\frac{6}{5}\right)$

(D) (2,7)

4 If the letters of the word WOOLONGONG was rearranged to form a 'word', how many unique arrangements are possible if no restrictions applied?

(A) 10!

(B) $\frac{10!}{2!2!2!}$

(C) $\frac{10!}{4!2!}$

(D) $\frac{10!}{4!2!2!}$

5 What is the expression for the general solutions of $\tan x = 1$?

(A) $n\pi + \frac{\pi}{4}$ (where n is an integer).

(B) $n\pi \pm \frac{\pi}{4}$ (where n is an integer).

(C) $2n\pi + \frac{\pi}{4}$ (where n is an integer).

(D) $2n\pi \pm \frac{\pi}{4}$ (where n is an integer).

6 Which of the following equates to $\int \cot x\, dx$?

(A) $\log_e x + c$

(B) $\log_e(\sin x) + c$

(C) $\log_e(\cos x) + c$

(D) $\log_e(\tan x) + c$

7 If $\cos x - \sqrt{3}\sin x \equiv R\cos(x + \alpha)$, which of the following represent the values of R and α?

(A) $R = 2, \alpha = \frac{\pi}{6}$

(B) $R = 2, \alpha = \frac{\pi}{3}$

(C) $R = \sqrt{2}, \alpha = \frac{\pi}{6}$

(D) $R = \sqrt{2}, \alpha = \frac{\pi}{3}$

8 What is this derivative of $y = \sin^{-1}\left(\frac{1}{x}\right)$?

(A) $\frac{1}{x\sqrt{x^2 - 1}}$

(B) $\frac{1}{\sqrt{x^2 - 1}}$

(C) $\frac{-1}{x\sqrt{x^2 - 1}}$

(D) $\frac{-1}{\sqrt{x^2 - 1}}$

9 A spherical balloon was slowly inflated. At the point where its radius is 2 cm, the rate of change of its radius is 3 cm/s. What is the rate of change of its volume $\frac{dV}{dt}$ at this point?

Note: Volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

(A) $\frac{dV}{dt} = 4\pi\,\text{cm}^3/\text{s}$

(B) $\frac{dV}{dt} = 12\pi\,\text{cm}^3/\text{s}$

(C) $\frac{dV}{dt} = 16\pi\,\text{cm}^3/\text{s}$

(D) $\frac{dV}{dt} = 48\pi\,\text{cm}^3/\text{s}$

10 If $\cos\theta = -\frac{3}{5}$ and $0 < \theta < \pi$, then $\tan\frac{\theta}{2}$ is equal to:

(A) $-\frac{1}{3}$ or -3

(B) $\frac{1}{3}$ or 3

(C) -2

(D) 2

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question on a NEW page on your OWN PAPER.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW page on your OWN PAPER.

(a) Solve for x: $\dfrac{3x+1}{x-3} \geq 1$. **3**

(b) Use the substitution $u = e^{\frac{x}{2}}$ to evaluate $\displaystyle\int \frac{e^{\frac{x}{2}}}{1+e^{x}}\,dx$. **3**

(c) Find the exact value of $\sin\left(2\cos^{-1}\dfrac{2}{3}\right)$. **2**

(d) Evaluate $\displaystyle\lim_{x \to 0} \frac{\sin \pi x}{x}$. **2**

(e) Find the term independent of x in the expansion of $\left(\dfrac{x}{3} - \dfrac{2}{x^2}\right)^{12}$. **3**

(f) The function $f(x) = \sin x + \cos x - x$ has a root near $x = 1.2$. Taking $x = 1.2$ as a first approximation, use one application of Newton's method to find a second approximation to the root. Give your answer correct to two decimal places. **2**

Question 12 (15 marks) Use a NEW page on your OWN PAPER.

(a) For the function: $y = 4\sin^{-1}3x - \pi$.

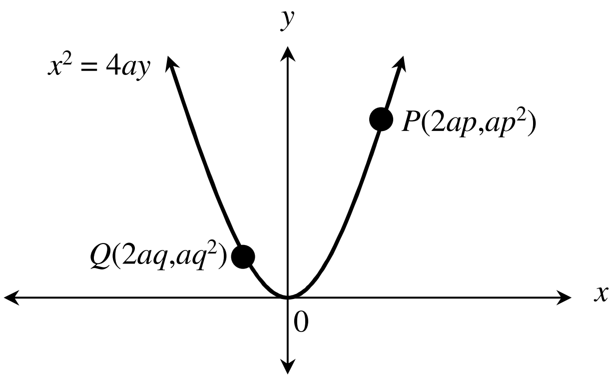
(i) State the function's domain and range. **2**

(ii) Hence, or otherwise, sketch the graph $y = 4\sin^{-1}3x - \pi$ on a number line, showing all key features. **2**

(b) (i) Find the area bound by the curve $y = \dfrac{2}{x-3}$ and the x-axis, between $x = 4$ and $x = 7$. **2**

(ii) Find the volume of the solid formed when the curve $y = \dfrac{2}{x-3}$ is rotated about the x-axis between $x = 4$ and $x = 7$. **2**

(c) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.



(i) Show that the midpoint, M, of the chord PQ is $\left(a(p+q), \dfrac{a(p^2+q^2)}{2}\right)$. **1**

(ii) If $pq = -2$, find the Cartesian equation of the locus of M. **2**

(d) A group of nine friends arrived at a restaurant.

(i) If they were to be seated around a circular table, how many possible arrangements are possible if:

(α) No restrictions are applied? **1**

(β) Three particular friends wanted to be seated together as a group? **1**

(ii) If they were to be seated around two circular tables, one with five seats and the other with four seats, how many possible arrangements are possible if no restrictions are applied? **2**

Question 13 (15 marks) Use a NEW page on your OWN PAPER.

(a) Use mathematical induction to prove for all integers $n \geq 1$: **3**

$$\frac{2\times 1}{2\times 3}+\frac{2^2\times 2}{3\times 4}+\frac{2^3\times 2}{4\times 5}+\ldots+\frac{2^n\times n}{(n+1)(n+2)}=\frac{2^{n+1}}{n+2}-1.$$

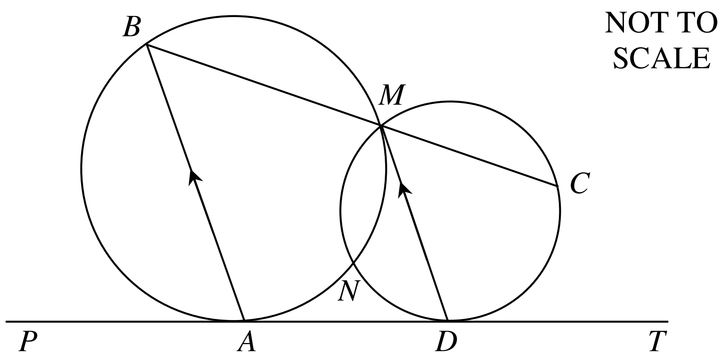
(b) The polynomial $P(x) = x^3 + 3x^2 - x - 4$ has roots α, β and γ.

(i) Find the value of $\alpha+\beta+\gamma$. **1**

(ii) Find the value of $\alpha^2+\beta^2+\gamma^2$. **1**

(iii) Find the value of $\alpha^3+\beta^3+\gamma^3$. **2**

(c) Two circles, one larger than the other, intersect at M and N. PT is a common tangent that meets the larger circle at A and the smaller circle at D. BM produced meets the smaller circle at C. $AB \parallel DM$.



Copy the diagram into your writing booklet.

(i) Prove that $ABCD$ is a cyclic quadrilateral. **2**

(ii) Prove that $AM \parallel DC$. **1**

(d) A particle moves along a straight line. Its displacement of x metres after t seconds is given by the formula:

$$x = 6\sin 2t.$$

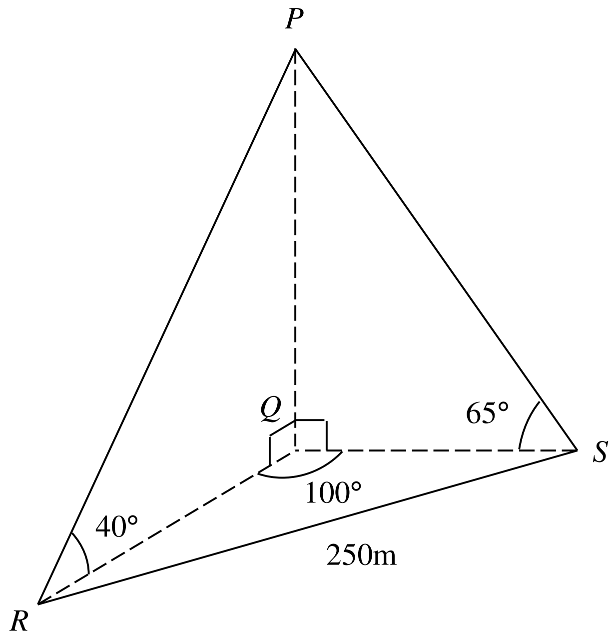
(i) Show that the particle moves with simple harmonic motion. **2**

(ii) Once in motion, when is the earliest the particle come to rest? **1**

(iii) Find the particle's maximum velocity. **2**

Question 14 (15 marks) Use a NEW page on your OWN PAPER.

(a) The diagram shown is a triangular pyramid where $\angle PRQ = 40°$, $\angle PSQ = 65°$, and $\angle RQS = 100°$. **3**



If the length of RS is 250m, find the length of PQ to one decimal place.

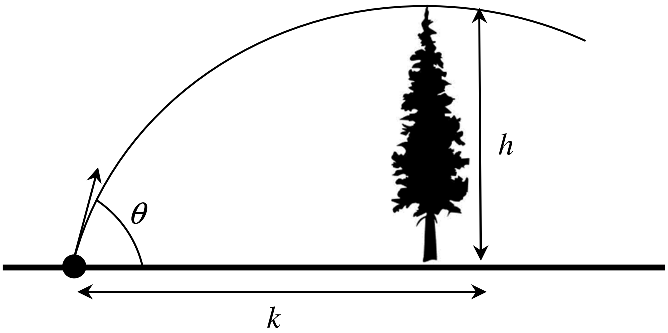
(b) Show that: $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$. **2**

(c) Given the identity $(1 + x)^n = {}^nC_0 + {}^nC_1 x^1 + {}^nC_2 x^2 + {}^nC_3 x^3 + ... + {}^nC_n x^n$ using the binomial theorem, show that: **4**

$$\frac{{}^nC_0}{1\times 2} + \frac{{}^nC_1}{2\times 3} + \frac{{}^nC_2}{3\times 4} + ... + \frac{{}^nC_n}{(n+1)(n+2)} = \frac{2^{n+2} - n - 3}{(n+1)(n+2)}$$

where n is a positive integer.

(d) A golfer was k metres away from the base of a tree h metres tall. To clear the tree, the golfer knew he had to chip the ball at an angle of θ with velocity V m/s such the ball cleared the tree at the maximum height of its projectile motion.



After t seconds, the horizontal (x) and vertical (y) displacements of the ball is given as follows (**DO NOT PROVE THESE**):

$$x = Vt\cos\theta \qquad \text{and} \qquad y = -\frac{gt^2}{2} + Vt\sin\theta$$

where gravity is g m/s^2.

(i) Show that the ball attains a maximum height when $t = \dfrac{V\sin\theta}{g}$. **2**

(ii) Show that $V^2 = \dfrac{g}{2h}\left(4h^2+k^2\right)$. **3**

(iii) Hence, or otherwise, show that $\theta = \tan^{-1}\left(\dfrac{2h}{k}\right)$. **1**

End of paper.