NEX

2021

**TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 100

(**Section I**) Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

(**Section II**) Pages 7 – 15

90 marks

- Attempt Questions 11 – 32
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

<u>Circle the BEST solution</u> below for Questions 1 – 10.

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1   What are the solutions for x in the equation $x^2 + x - 72 = 0$?

    (A) $x = 72, -1$

    (B) $x = 8, -9$

    (C) $x = 7, -8$

    (D) $x = \dfrac{-1 \pm \sqrt{73}}{2}$

2   What is the limiting sum of the series $24 + 6 + 1.5 + \ldots$

    (A)   32

    (B)   36

    (C)   40

    (D)   42

3   Given that $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \mid B) = 0.25$, what is the value of $P(A \cap B)$?

    (A)   0.1

    (B)   0.15

    (C)   0.35

    (D)   0.75

4   The following diagram shows an aerial view of a local park. The measurements shown are taken at equally spaced intervals perpendicular to the 90m frontage of the park.
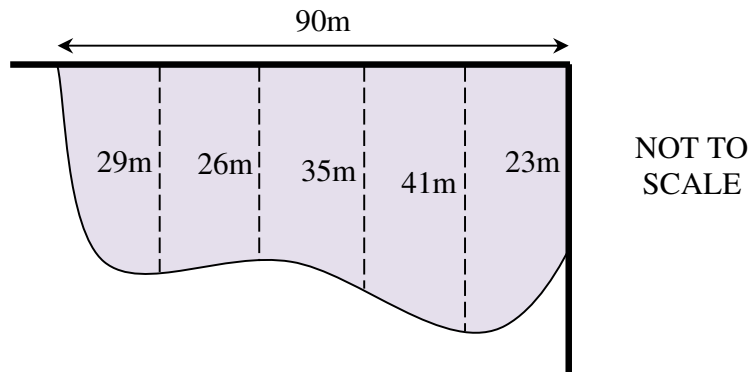
What is the approximate area of the park using the trapezoidal rule?

(A)   Area = 2565 m^2

(B)   Area = 2772 m^2

(C)   Area = 12825 m^2

(D)   Area = 13860 m^2

5   What is the derivative of $\log_e\left(\log_e(5x)\right)$?

(A) $-\dfrac{1}{5x}$

(B) $\dfrac{1}{x}$

(C) $\dfrac{1}{x\log_e(5x)}$

(D) $\dfrac{5x}{\log_e(5x)}$

6   What is the area of the shaded segment in the diagram?

40°

6 cm
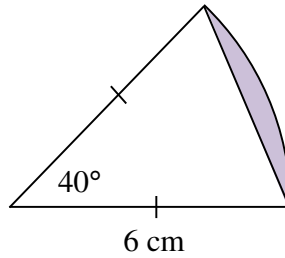
(A)   0.996 cm²

(B)   1.222 cm²

(C)   11.570 cm²

(D)   12.566 cm²

7   The table below shows the present value of an annuity paying $1 at the end of each year at various interest rates.

| End of Year	Present Value of a $1 Annuity				
	Interest Rate (per period)				
	5.00%	5.50%	6.00%	6.50%	7.00%
1	0.9524	0.9479	0.9434	0.9390	0.9346
2	1.8594	1.8463	1.8334	1.8206	1.8080
3	2.7232	2.6979	2.6730	2.6485	2.6243
4	3.5460	3.5052	3.4651	3.4258	3.3872
5	4.3295	4.2703	4.2124	4.1557	4.1002
6	5.0757	4.9955	4.9173	4.8410	4.7665
7	5.7864	5.6830	5.5824	5.4845	5.3893
8	6.4632	6.3346	6.2098	6.0888	5.9713
9	7.1078	6.9522	6.8017	6.6561	6.5152
10	7.7217	7.5376	7.3601	7.1888	7.0236

What the is present value of an annuity that pays $8500 each year over nine years that accrues interest at 5.50% p.a.?

(A) $13,762.30

(B) $53,843.80

(C) $59,093.70

(D) $60,406.50

8   By applying the interquartile formula for determining outliers, which of the following numbers can be considered an outlier?

$$35 \quad 42 \quad 29 \quad 34 \quad 75 \quad 21 \quad 31 \quad 48$$

(A)   21 and 75.

(B)   75 only.

(C)   21 only.

(D)   There are no outliers.

9   The following diagram shows the sketch of $y = g(x)$.
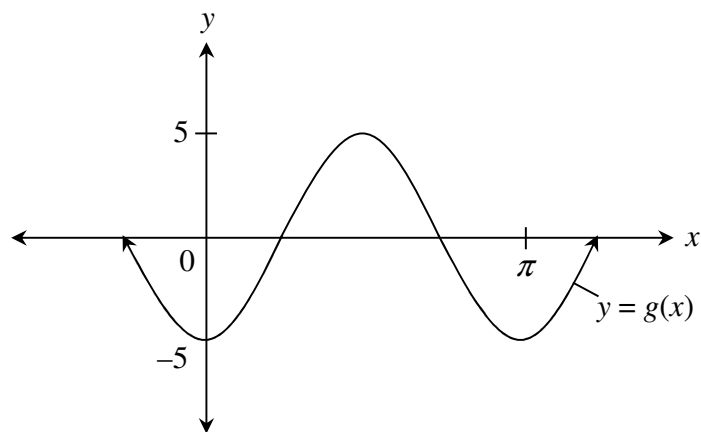
Which of the following describes the transformation of $y = \cos x$ to $y = g(x)$?

(A)   Vertical dilation by a factor of 2; *then* horizontal dilation by factor of 5.

(B)   Reflection about the x-axis; *then* vertical dilation by a factor of 5.

(C)   Vertical dilation by a factor of $\dfrac{1}{5}$; *then* horizontal dilation by factor of 2.

(D)   Reflection about the x-axis; vertical dilation by a factor of 5; *then* horizontal dilation by factor of $\dfrac{1}{2}$.
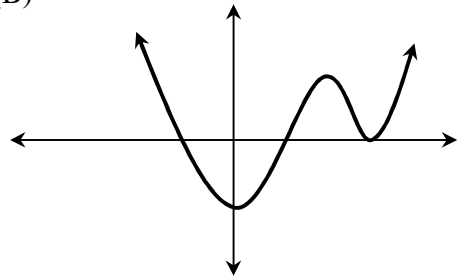
10   Consider the following function $y = f(x)$:
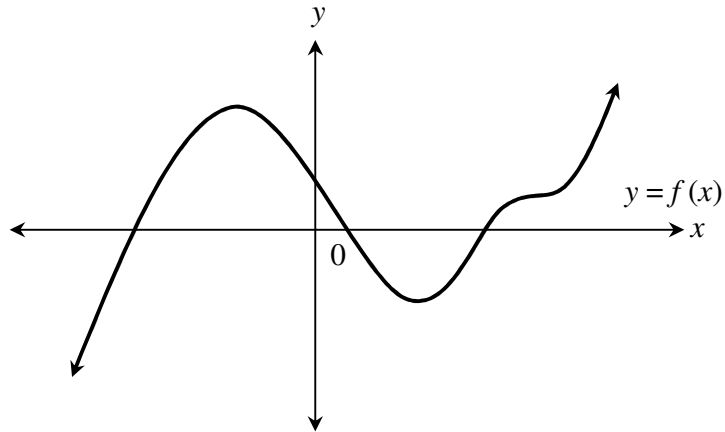
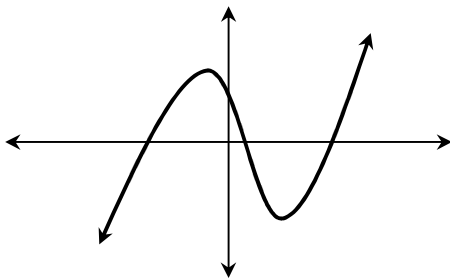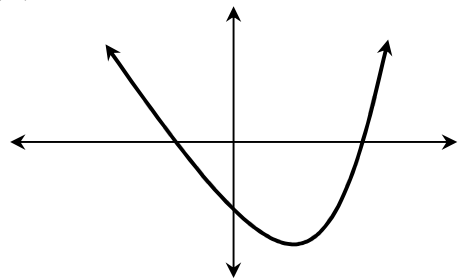Which of the following represents the sketch of $y = f'(x)$?

(A)
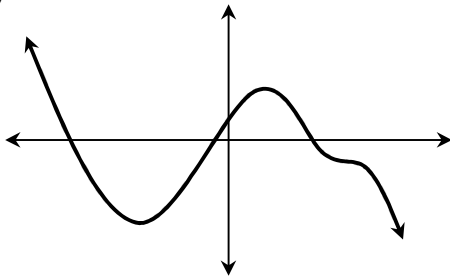
(B)

(C)

(D)

Section II

90 marks
Attempt Questions 11–32

In Questions 11–32, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Solve for x, expressing your solution in set notation: $\dfrac{x+2}{3x-5} \geq 0.$ **2**

Question 12

For the series $36 - 12 + 4 - \dfrac{4}{3} + \dots$

(a)   Show that the series follows a geometric progression. **1**

(b)   Find the eight term in the series. **1**

(c)   Find the sum of the first eight terms in the series. **1**

Question 13

Differentiate the following with respect to x:

(a) $y = \dfrac{7}{e^3}.$ **1**

(b) $y = \tan 2x.$ **1**

(c) $y = (\cos 4x + 5)^9.$ **2**

(d) $y = \log_e \left(\dfrac{4x-1}{3x+1} \right).$ **2**

Question 14

Find the equation of the tangent to the curve $y = \left(x^2 + 1\right)^3$ at the point where $x = 1$. **3**

Question 15

The probability distribution of a discrete random variable X is summarised in the following table:

x	-1	2	3	7	9
$P(X = x)$	0.4	$4a$	0.13	a	0.07

where a is constants.

(a)   Find the value of a. **2**

Using part (a),

(b)   Find the expected value of X. **1**

(c)   Find the variance of X. **1**

Question 16

Solve for x: $4^x - 17(2^x) + 16 = 0$ **3**

Question 17

Given that $\log_x a = 9.51$ and $\log_x b = 11.47$, find the value of $\log_x \sqrt{ab}$. **2**

Question 18

Prove the identity: $\dfrac{1 + \cot x}{\operatorname{cosec} x} - \cos x = \dfrac{\sec x}{\tan x + \cot x}$. **3**

Question 19

Find:

(a) $\displaystyle\int \frac{6}{\sqrt[5]{x}}\, dx$.   **1**

(b) $\displaystyle\int \frac{x^2}{x^3+2}\, dx$.   **1**

(c) $\displaystyle\int 4\sin\left(\pi x\right)e^{\cos\left(\pi x\right)}\, dx$.   **2**

Question 20

If $\sec\theta = \dfrac{7}{3}$ and $\tan\theta < 0$, find the exact value of $\sin\theta$.   **2**

Question 21

A pod of pelicans was monitored on a small island as part of a scientific study. The rate at which the pod's population (P) increases at is given by the equation:

$$\frac{dP}{dt} = kP.$$

where k is a positive constant.

(a)   Verify that $P = Ae^{kt}$ is a solution to the differential equation where A is the initial population of pelicans observed.   **1**

(b)   At the start of the study, the pelican population totalled 820. After 150 days of monitoring, the pelican population totalled 1190.

       (i)   Find the value of k to three significant figures.   **2**

       (ii)   Using part (ii), estimate the pelican population after 350 days from the start of the study. Round your solution to the nearest whole number.   **1**
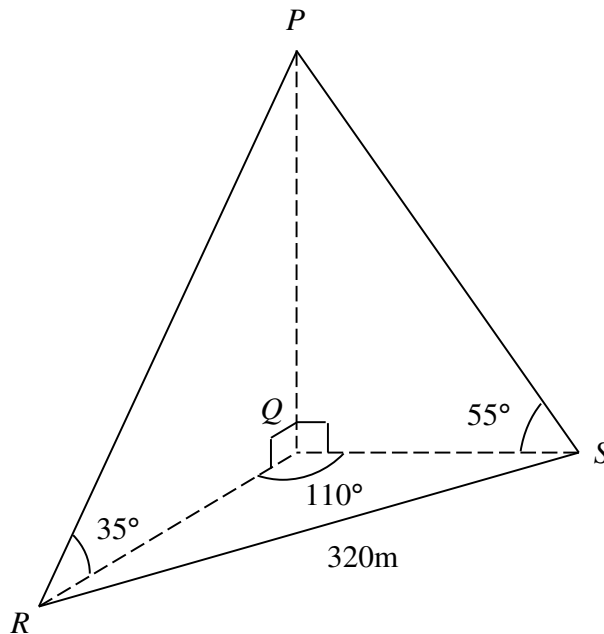
Question 22

A particle moves along a straight line where its displacement x metres after time t seconds is given by the formula:

$$x = 3 - 5\cos2t.$$

(a)   In terms of t, find an expression for the particle's velocity v and acceleration a.   **2**

(b)   Once in motion, when does the particle first come to rest?   **1**

(c)   What is the maximum displacement of the particle?   **1**

(d)   What is the maximum velocity of the particle?   **1**

Question 23

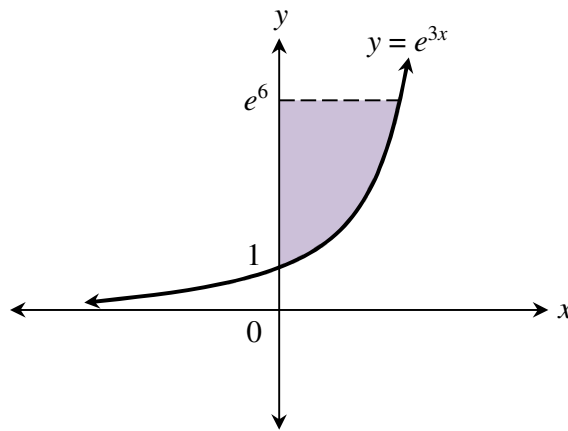The diagram shown is a triangular pyramid where $\angle PRQ = 35°$, $\angle PSQ = 55°$ and $\angle RQS = 110°$.   **3**

If the length of RS is 320m, find the length of PQ to one decimal place.

Question 24

Evaluate: $\displaystyle\int_{1}^{2} \frac{4x-3}{6x^2-9x+4}\,dx$ **2**

Question 25

The following diagram shows the area enclosed between the curve $y = e^{3x}$, the y-axis and   **3**
the line $y = e^6$.

Find the shaded area.

Question 26

In a coastal town the temperature during the summer season broadly followed a normal   **3**
distribution. Approximately 16% of the days had temperatures below or equal to 24.9°C,
while 2.5% of the days had temperatures above 29.1°C.

Using the empirical rule, find the <u>mean</u> and <u>standard deviation</u> for the temperature observed over
the summer season for this coastal town.

Question 27

Consider the functions $f(x) = x^2 - 9$ and $g(x) = \sqrt{x}$.   **2**

Find the domain and range of the composite function $f\big(g(x)\big)$.

Question 28

For the curve $y = xe^x + e^x$,

(a)   Find any x-intercepts and y-intercepts.   **2**

(b)   Find the stationary points and determine their nature.   **3**

(c)   Find any points of inflection.   **2**

(d)   Hence, or otherwise, sketch the curve $y = xe^x + e^x$ on a number plane, showing all key features.   **2**

Question 29

A cylinder is inscribed in a sphere with radius of 5 cm. The height of the cylinder is h cm and the radius is x cm, as shown in the diagram.
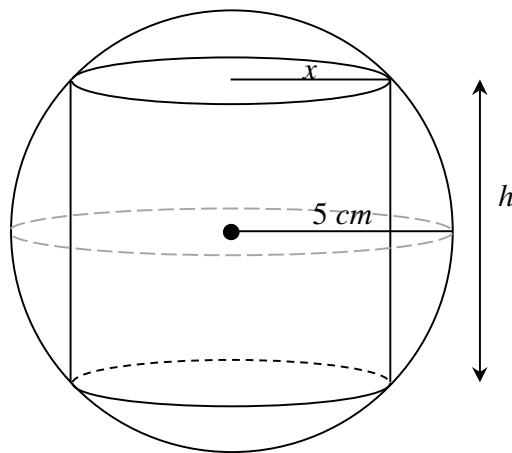
(a)   Show that the volume, V, of the cylinder is given by $V = \pi x^2 \sqrt{100 - 4x^2}$ cm³.   **2**

(b)   Find the value of x for which the volume of the cylinder is a maximum.   **3**

Question 30

Insure-me-now is a company that specialises in providing car insurance to drivers. The insurance premium paid by the drivers (i.e. the price of the car insurance) may change each year depending on the number of insurance claims experienced.

The following table tracks the average increase in premium (*P*) for *Insure-me-now*'s car insurance product along with a summary of the number of insurance claims (*C*) made by drivers in the previous year.

Year	Number of insurance claims made in previous year (*C*)	Average increase in car insurance premium in dollars (*P*)
2011	624	6.80
2012	256	3.15
2013	1319	8.90
2014	552	4.70
2015	125	2.75
2016	334	1.85
2017	985	11.60
2018	1012	19.45
2019	681	7.20
2020	809	8.85

By using the table above, and rounding your solution to two decimal places where required:

(a)   Determine the mean and standard deviation of the average price increase each year,   **2**

(b)   By applying the *method of least squares regression* to the data in the table above,

    (i)   State the dependent variable.   **1**

    (ii)   Determine the value of Pearson's correlation coefficient (*r*).   **1**

    (iii)   Find the 'line of best fit', stating your solution as $P = \boxed{} + \boxed{} \times C$.   **2**

    (iv)   Using your solution to part (iii), estimate the average price increase in a year where 1550 claims were made.   **1**

Question 31

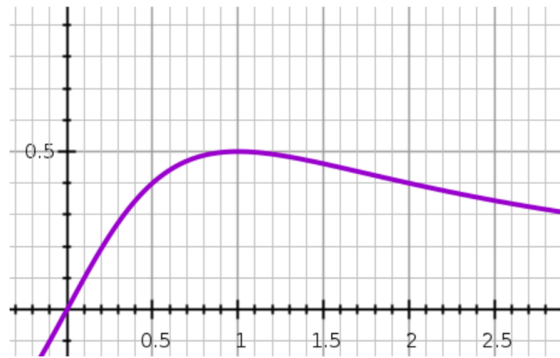(a)   Find $\int \dfrac{x}{x^2+1}\,dx$.   1

(b)   A continuous random variable X has a probability density function:

$$f(x) = \begin{cases} \dfrac{x}{x^2+1} & 0 \le x \le m \\ 0 & \text{otherwise} \end{cases} \qquad \text{where } m \in \mathbb{R}.$$

The graph of the function $y = \dfrac{x}{x^2+1}$ is shown below.

(i)   Find the value of m.   2

(ii)   Find the value of $P(0.5 \le X \le 0.7)$, leaving your answer in exact form.   2

(iii)   Find the mode of X.   2

Question 32

Poi takes out a $70,000 loan from a bank offering a reducible interest rate on of 6% per annum (compounded annually). Each year at p regular intervals, where $p \in \{1, \ 2, \ 4, \ 6, \ 12\}$, Poi makes repayments of $M at the start of each period. The loan is to be repaid over 10 years.

The amount owning on the loan after n periods is denoted by A_n.

(a)   Show that the amount owing after the third repayment is made is: **2**

$$A_3 = 70000\left(1.005^{\frac{12}{p}}\right)^3 - M\left(1 + 1.005^{\frac{12}{p}} + \left(1.005^{\frac{12}{p}}\right)^2\right).$$

(b)   Show that the amount of each repayment is given by: **3**

$$M = 70000\left(1.005^{120}\right)\left(\frac{1.005^{\frac{12}{p}} - 1}{1.005^{120} - 1}\right).$$

To the nearest cent,

(c)   Find the value of each repayment made by Poi if the repayments are made quarterly.   **1**

(d)   How much would Poi save over the term of the loan if he chose to make monthly repayments rather than quarterly repayments?   **2**

End of paper.